Semantics for Counterpart-based Temporal Logics

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Contribution and Outline

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5. Conclusion and future work

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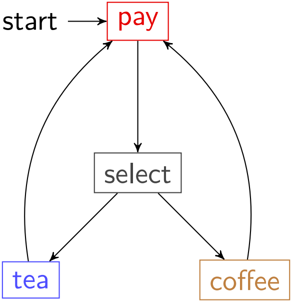

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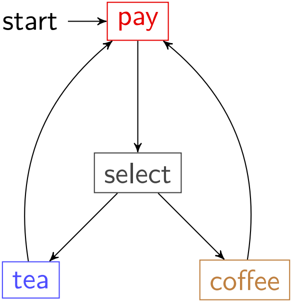

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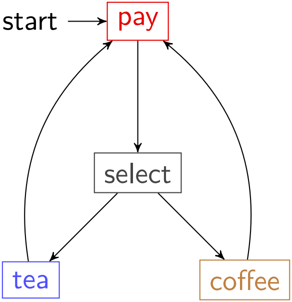

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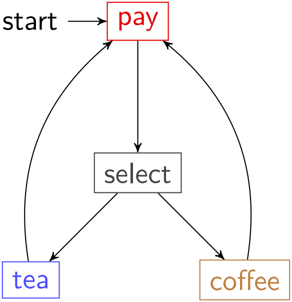

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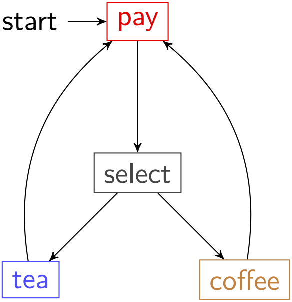

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3. Use a program to *check* that the *model* **satisfies** the formula

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- *Yes!* Using **counterpart models** and quantified temporal logics

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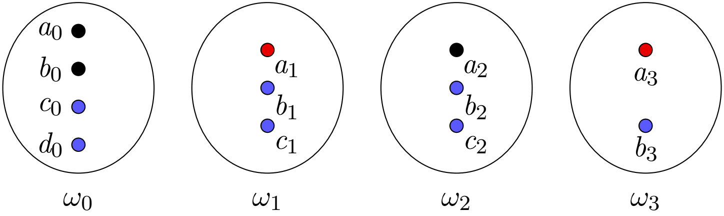

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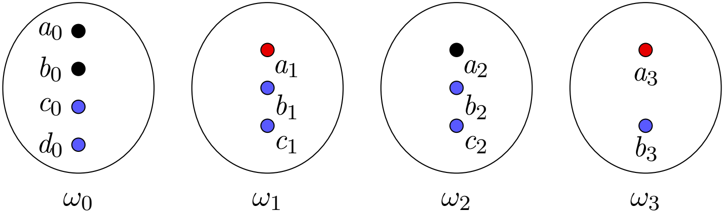

How do we represent transitions?

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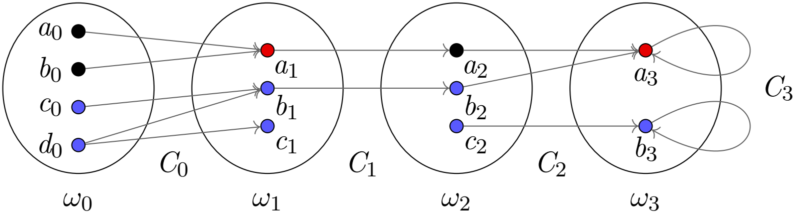

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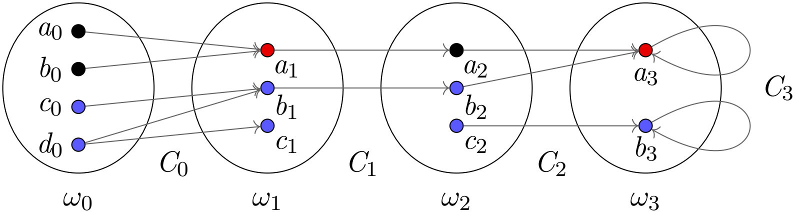

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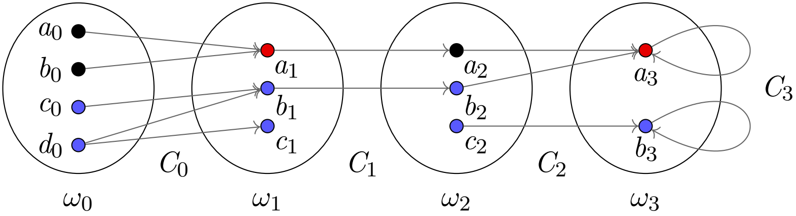

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- We call these sequences of worlds and relations **counterpart traces**

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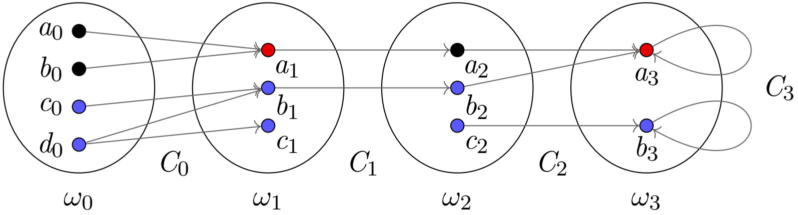

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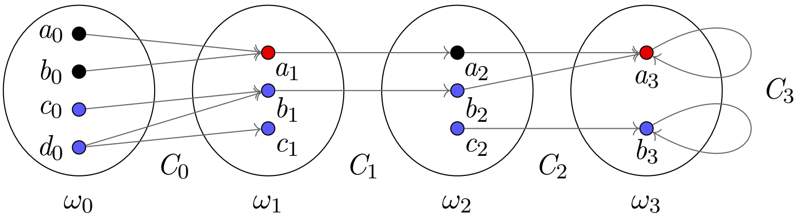

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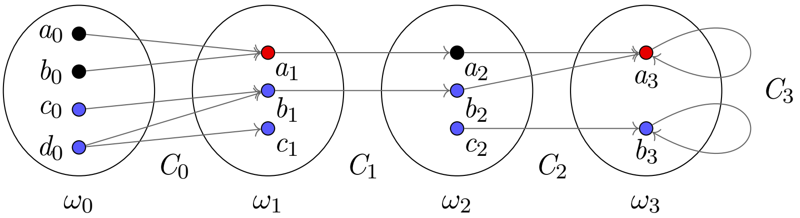

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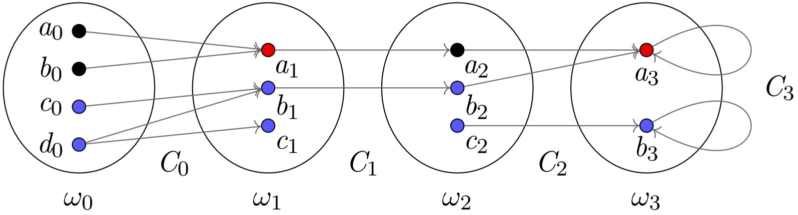

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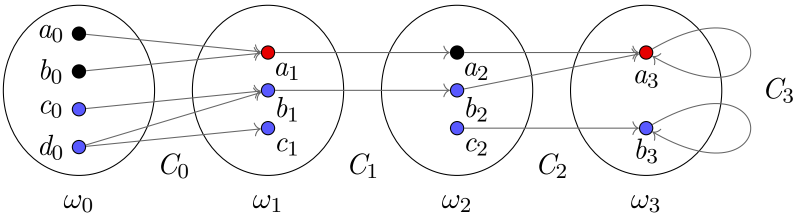

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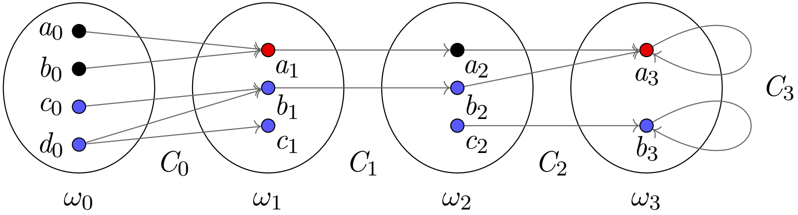

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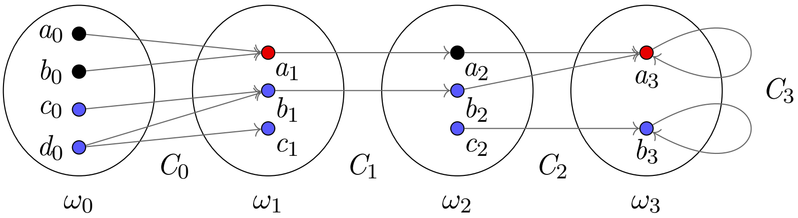

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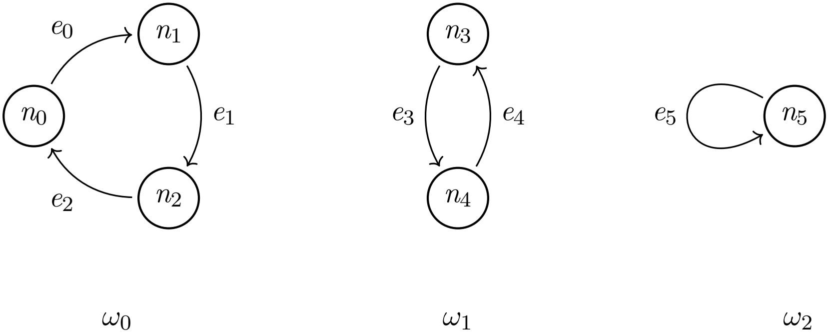

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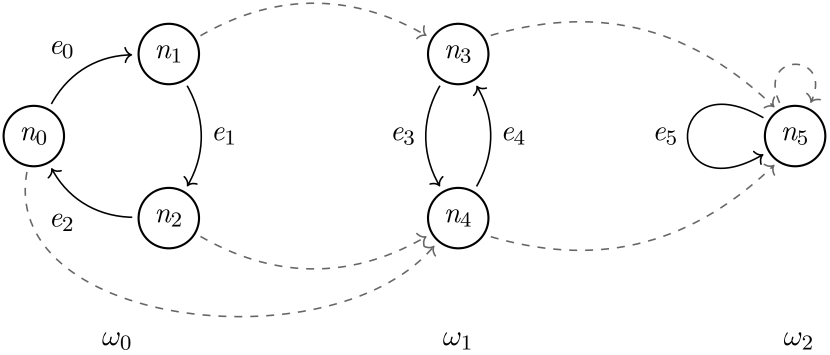

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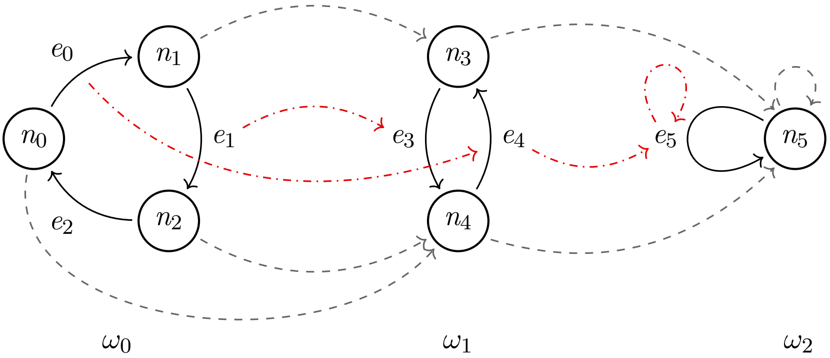

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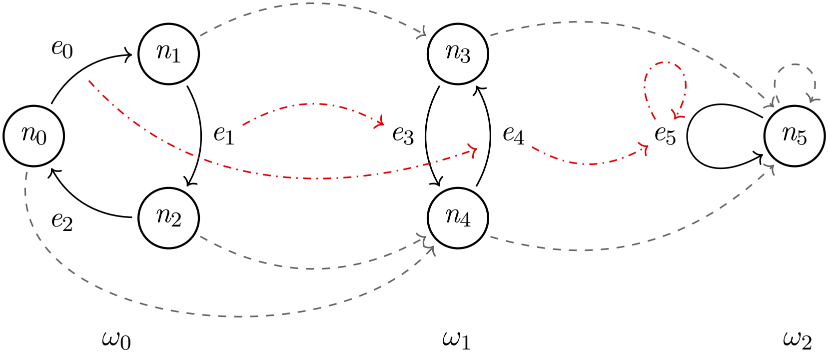

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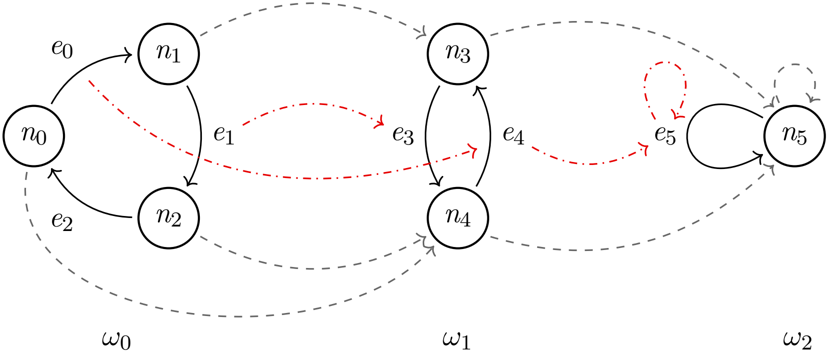

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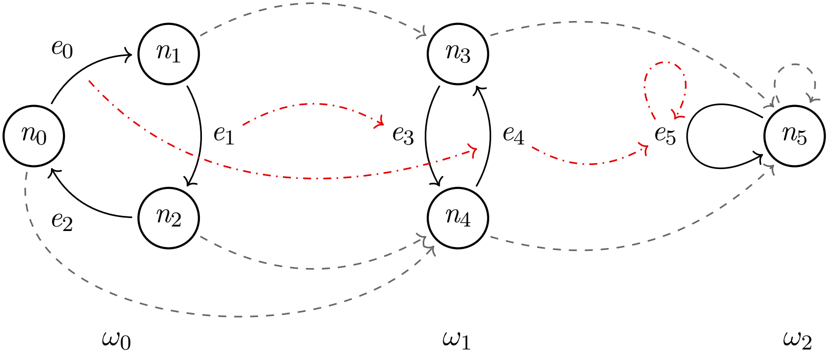

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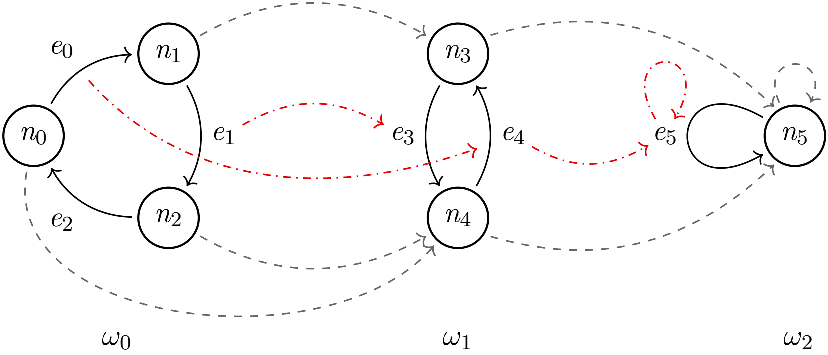

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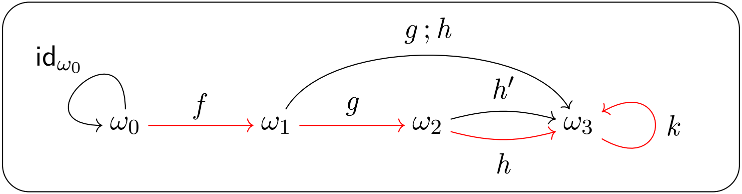
- The *relational presheaf* assigns worlds and counterpart relations to states

Relational presheaves – Example



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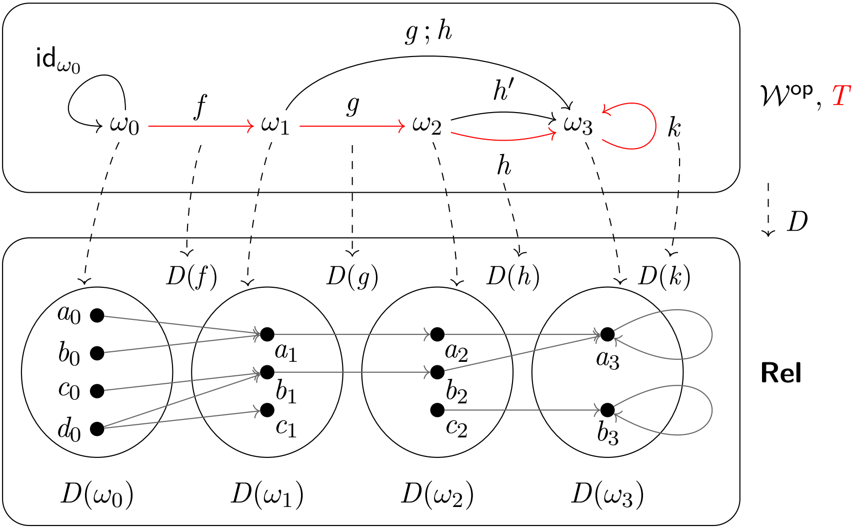

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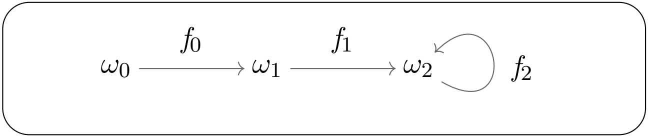

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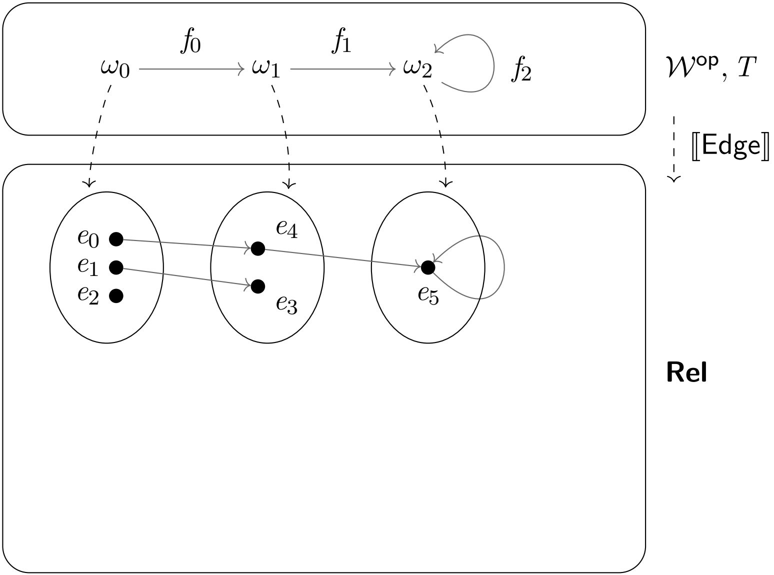

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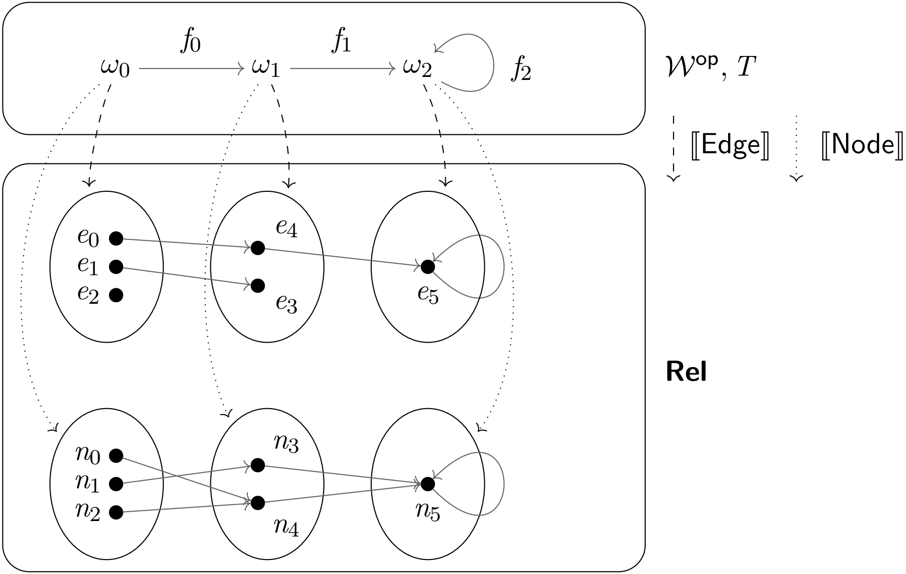

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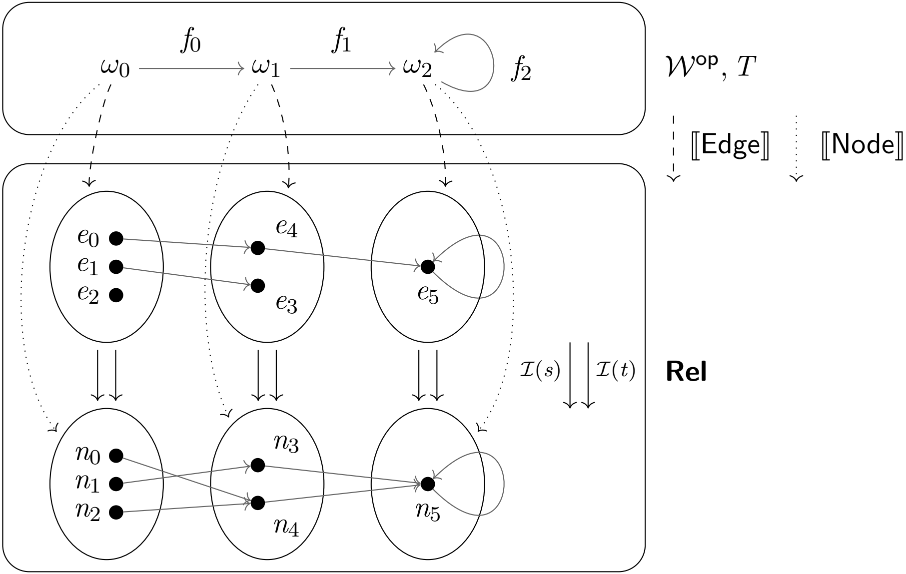

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- The *theorem* has not been formalized, but the *construction* is!

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⇒ Directly prove another formula instead of working with contradiction

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⇒ LTL-like expansion laws break down in the case of relations!

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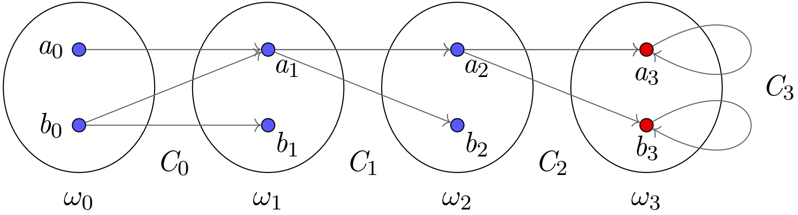

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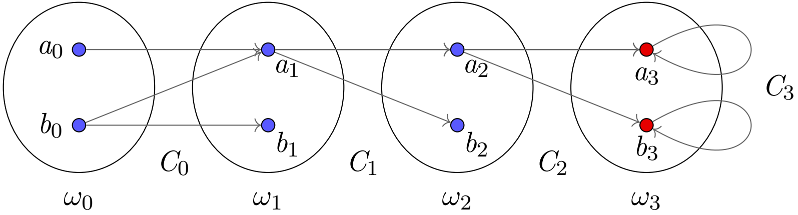

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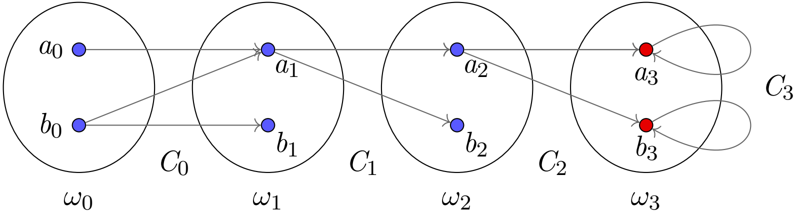

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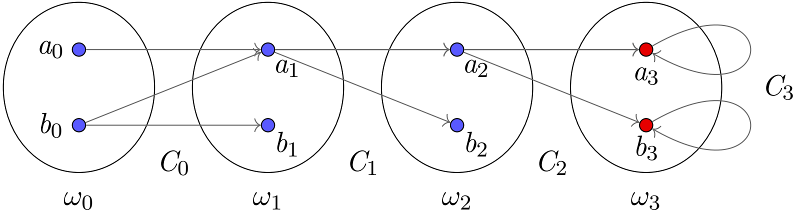

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- $\neg(\phi_1 \mathsf{Until} \phi_2) \equiv (\neg\phi_2)\mathsf{WUntilF}(\neg\phi_1 \wedge \neg\phi_2)$
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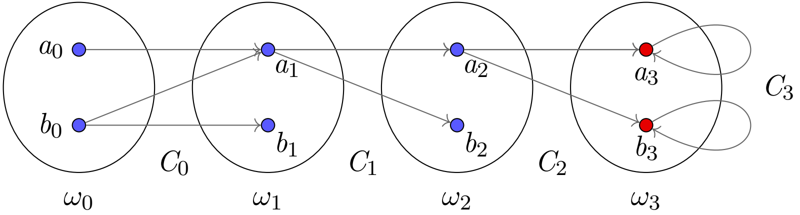
Positive normal forms for QLTL

- PNF for QLTL:

$$\psi := \mathsf{true} \mid x = y \mid P(x), \quad \phi := \psi \mid \neg\psi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \exists x.\phi \mid \forall x.\phi$$

$$\mid \mathsf{Next}(\phi) \mid \phi_1 \mathsf{Until} \phi_2 \mid \phi_1 \mathsf{WUntil} \phi_2 \mid \underline{\mathsf{NextF}}(\phi) \mid \phi_1 \underline{\mathsf{UntilF}} \phi_2 \mid \phi_1 \underline{\mathsf{WUntilF}} \phi_2$$



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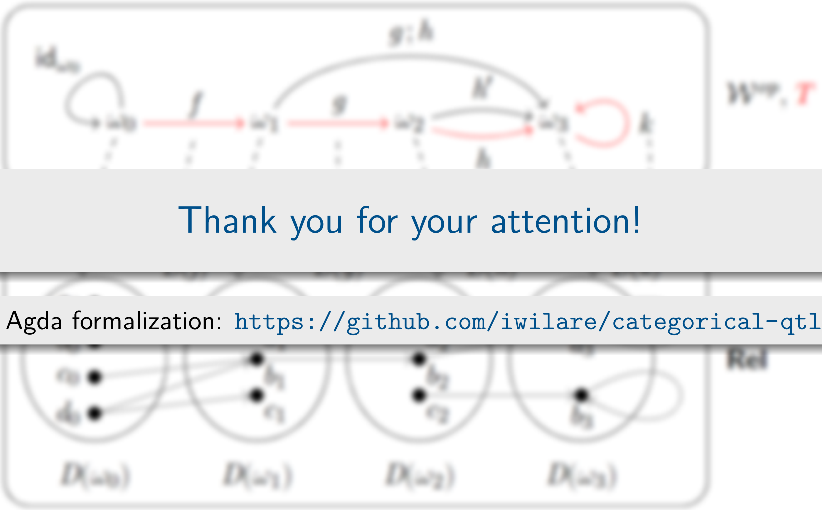

Thank you for your attention!

Agda formalization: `https://github.com/iwilare/categorical-qtl`