Categorical Semantics for Counterpart-based Temporal Logics in Agda

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Contribution and Outline

In this work we present the categorical semantics of a counterpart-based temporal logic, and formalize it using the proof assistant Agda along with results on its positive normal form.

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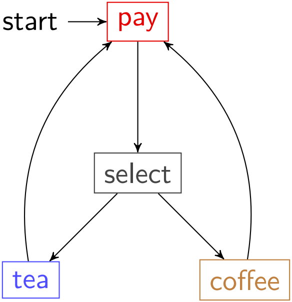

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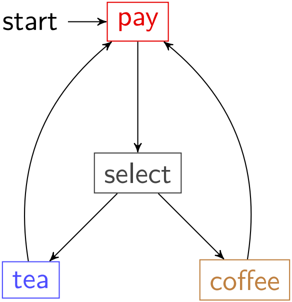

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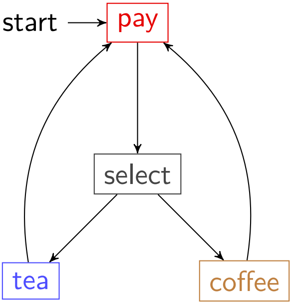

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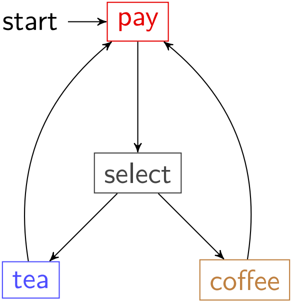

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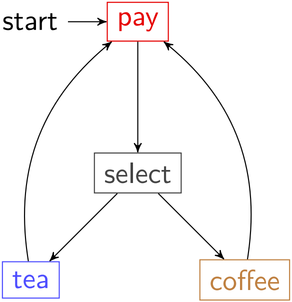

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3. Use a program to *check* that the *model* **satisfies** the formula

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- *Yes!* Using **counterpart models** and quantified temporal logics

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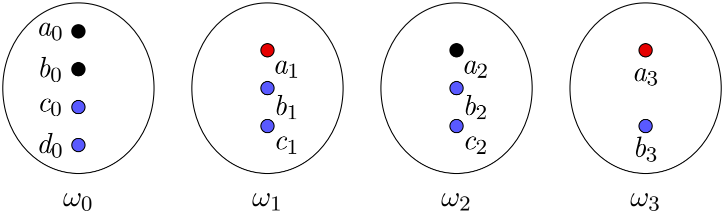

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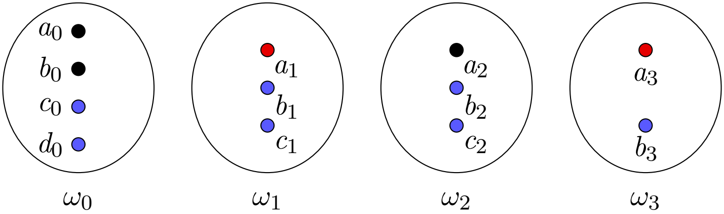

How do we represent transitions?

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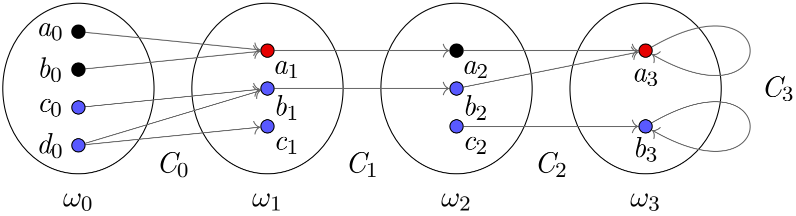

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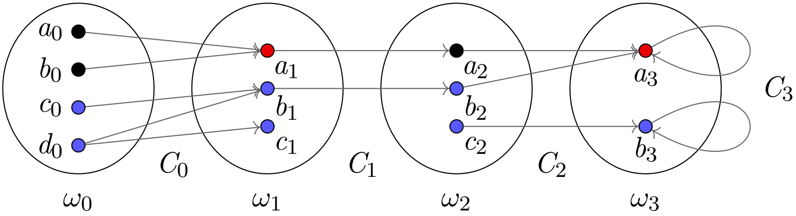

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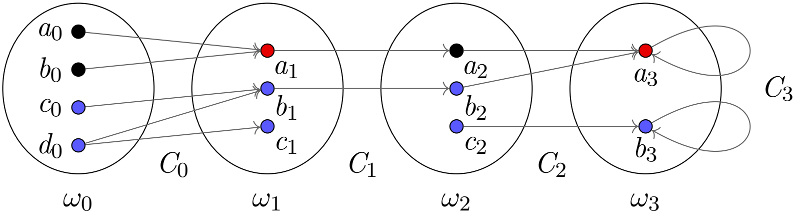

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- We call these sequences of worlds and relations **counterpart traces**

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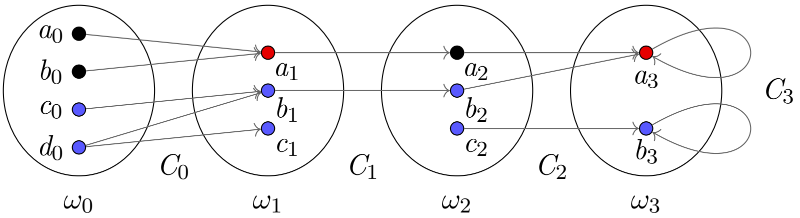

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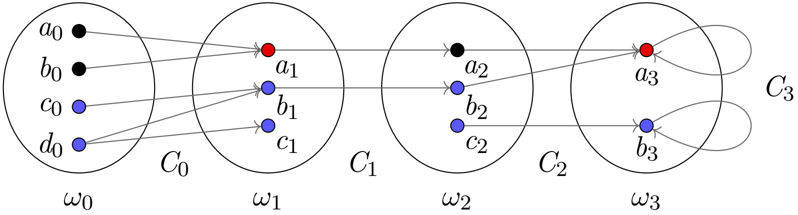

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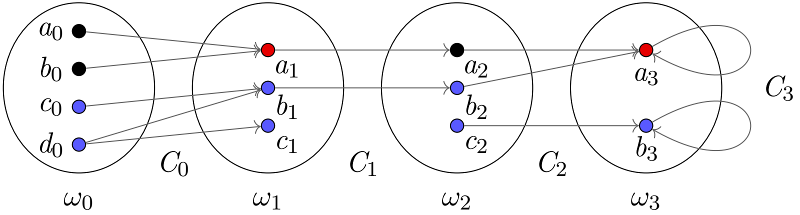

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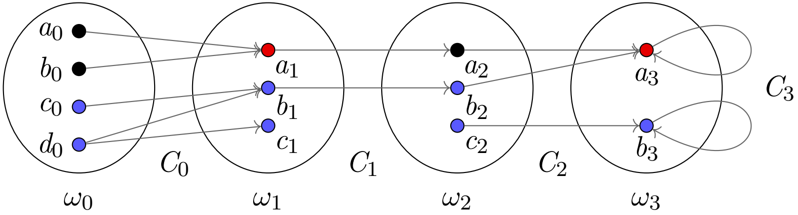

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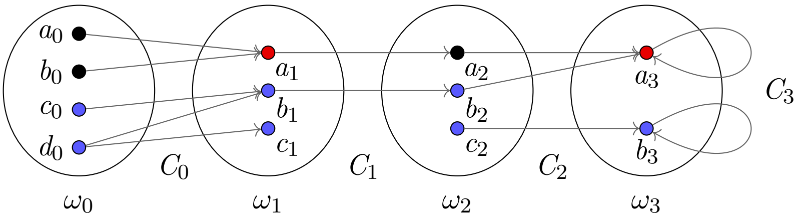

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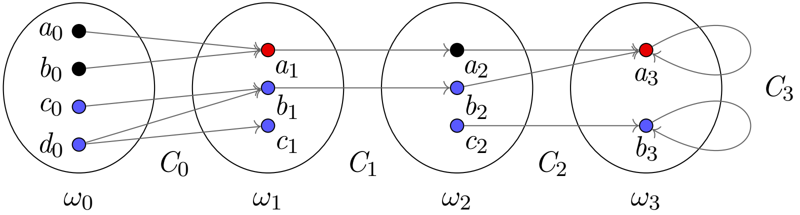

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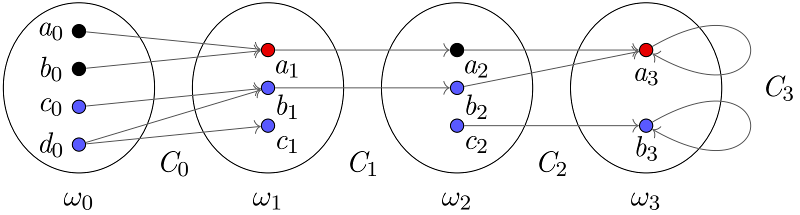

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- The *relational presheaf* assigns worlds and counterpart relations to states
- The *temporal structure* identifies the one-step transitions of the model

Relational presheaves – Example

$$
\begin{tikzcd}
\omega_0 \arrow[loop left, "\mathsf{id}_{\omega_0}"] \arrow[r, "f"] & \omega_1 \arrow[r, "g"] \arrow[rr, bend left=40, "g\,;h"] & \omega_2 \arrow[r, bend left, "h'"] \arrow[r, bend right, "h"'] & \omega_3 \arrow[loop right, "k"]
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\qquad \mathcal{W}^{op},\ T
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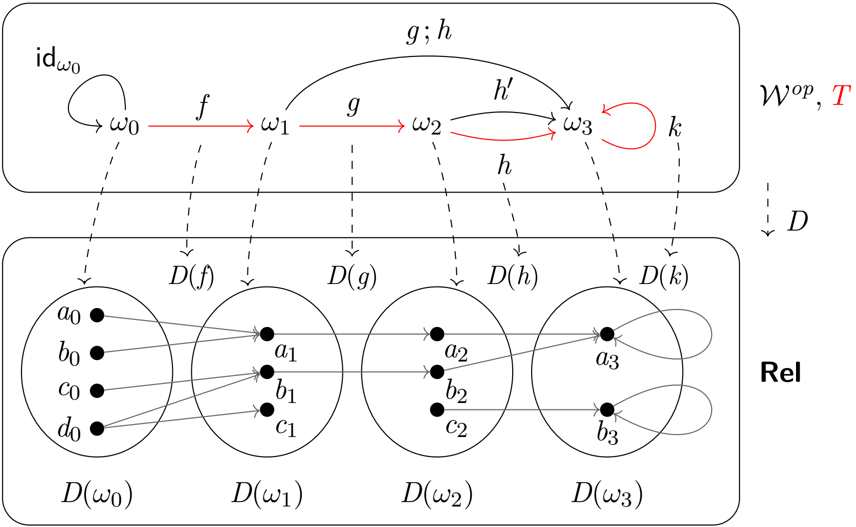

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- The *theorem* has not been formalized, but the *construction* is!

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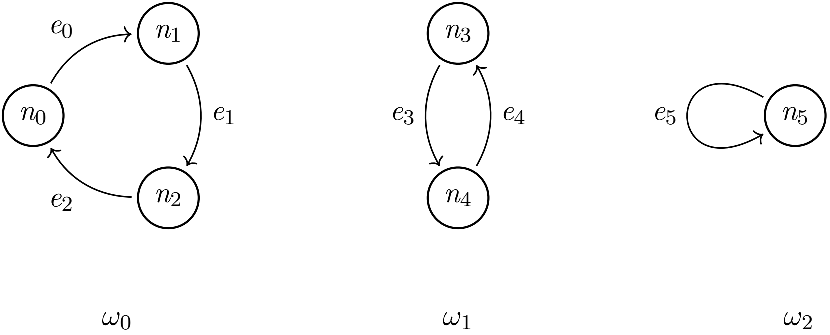

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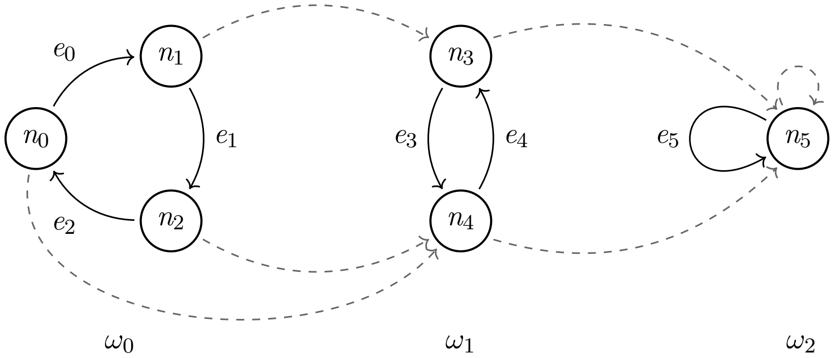

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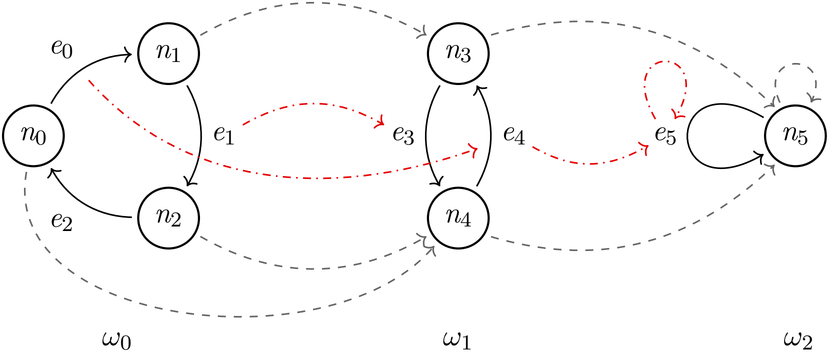

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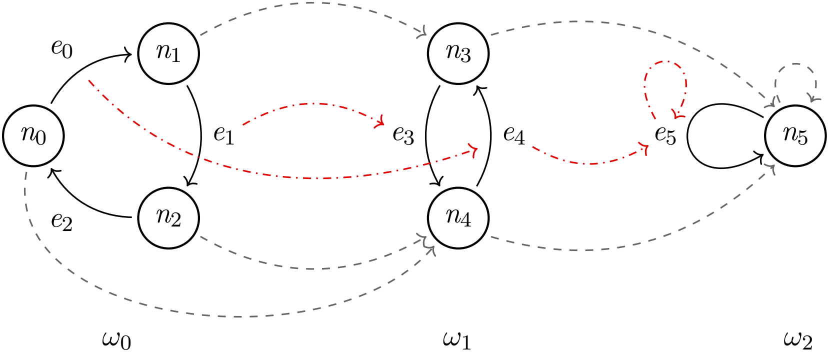

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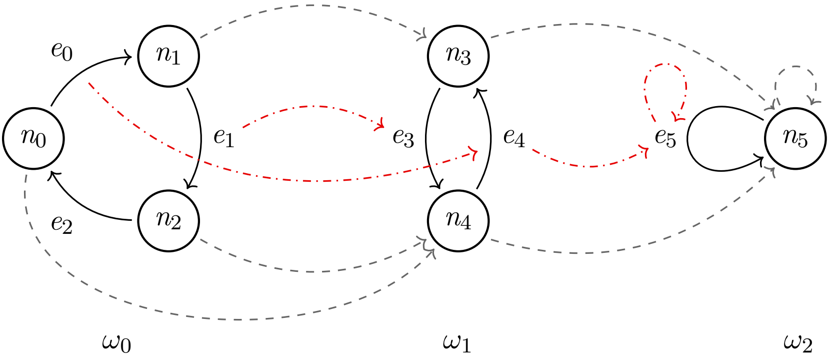

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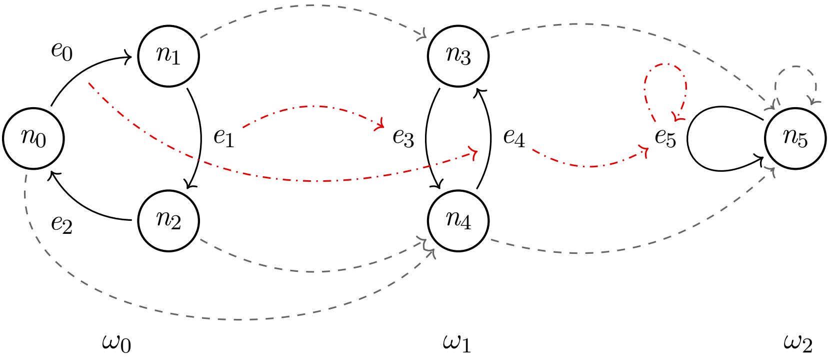

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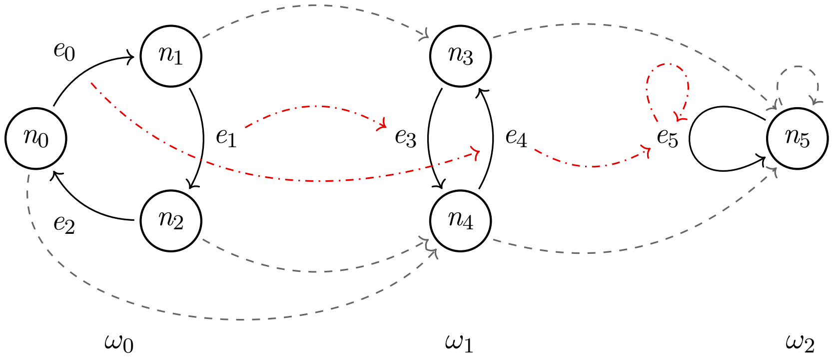

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$\mathcal{W}^{op}$, T

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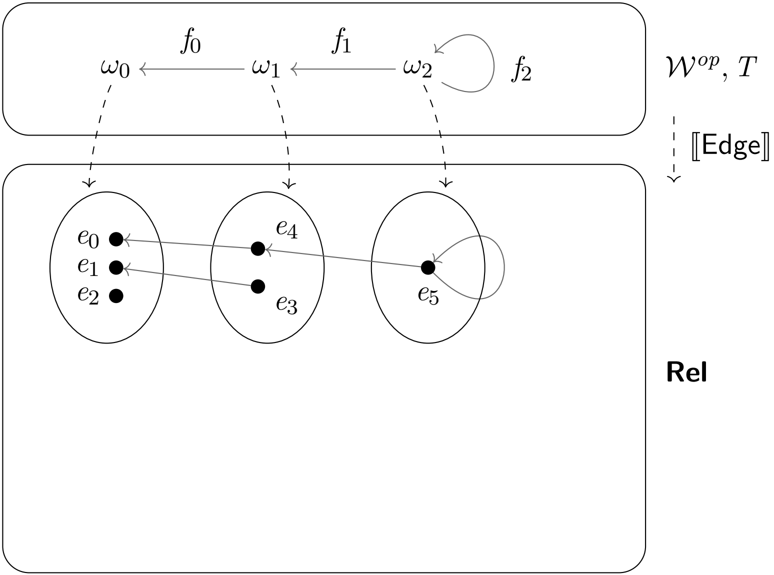

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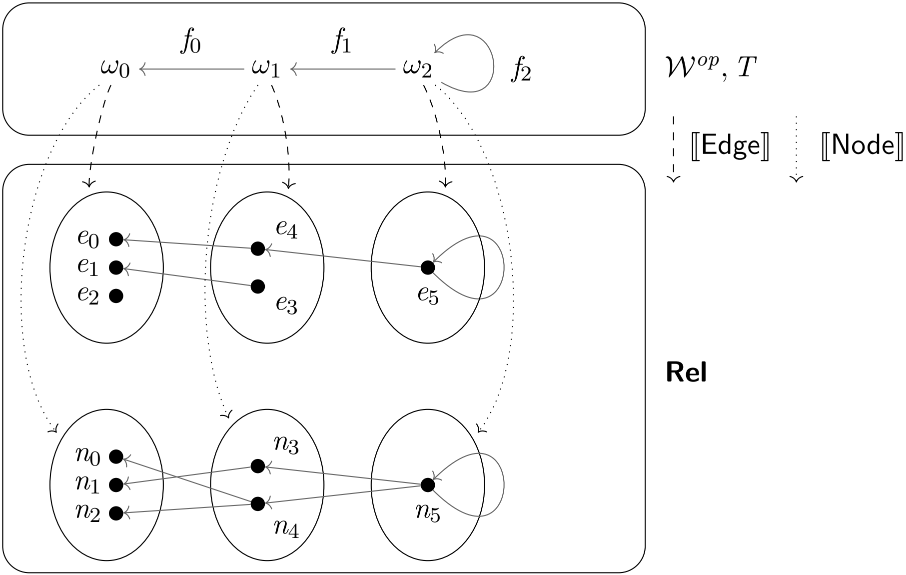

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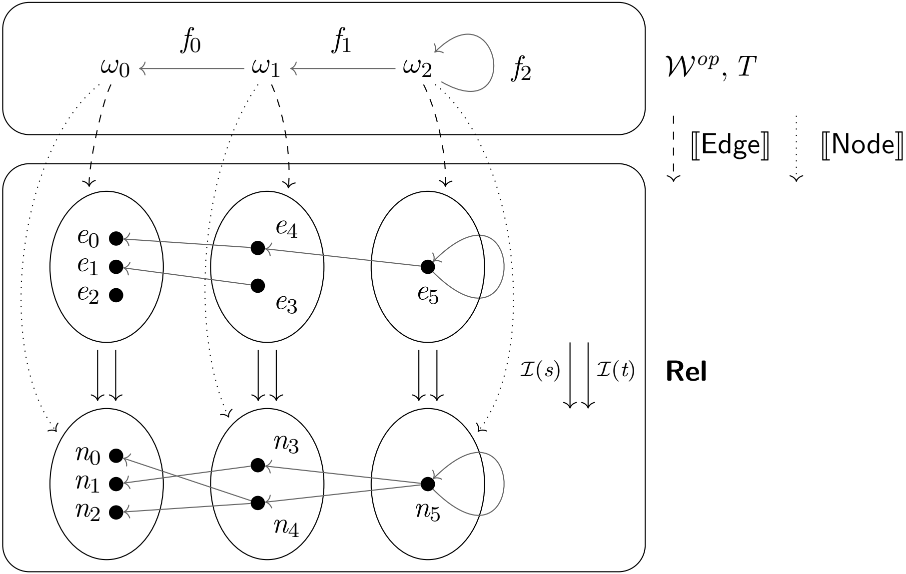

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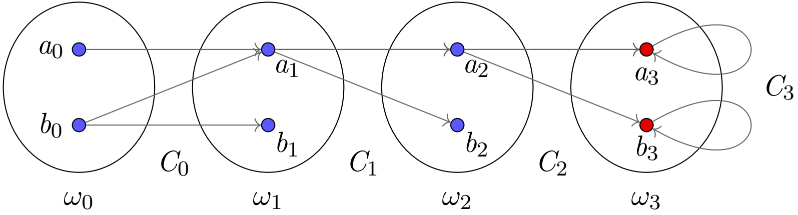

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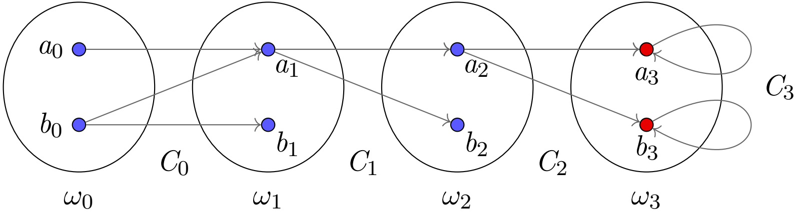

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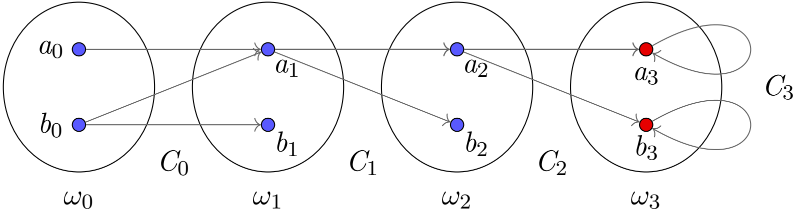

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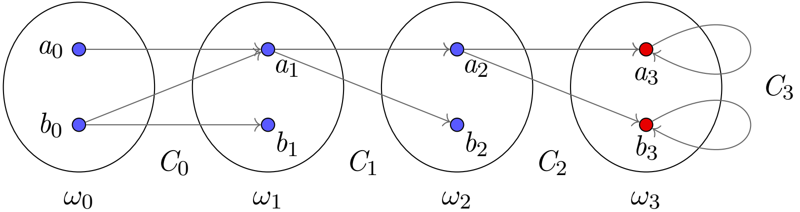

- $\neg\text{Next}(\phi) \equiv \text{NextF}(\neg\phi)$
- $\neg(\phi_1 \text{Until} \phi_2) \equiv (\neg\phi_2)\text{Then}(\neg\phi_1 \wedge \neg\phi_2)$
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- $b_0 \vDash_{\omega_0} \text{NextF}(\text{Blue}(x))$
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Positive normal forms for QLTL

- PNF for QLTL:

$$\psi := \text{true} \mid x = y \mid P(x), \quad \phi := \psi \mid \neg\psi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \exists x.\phi \mid \forall x.\phi$$
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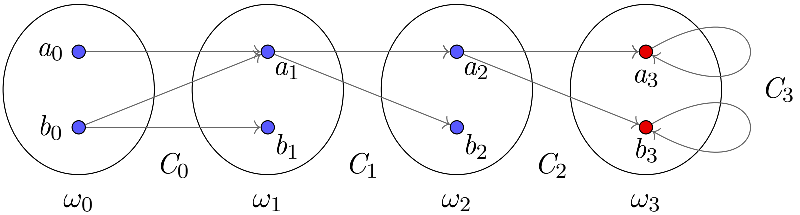

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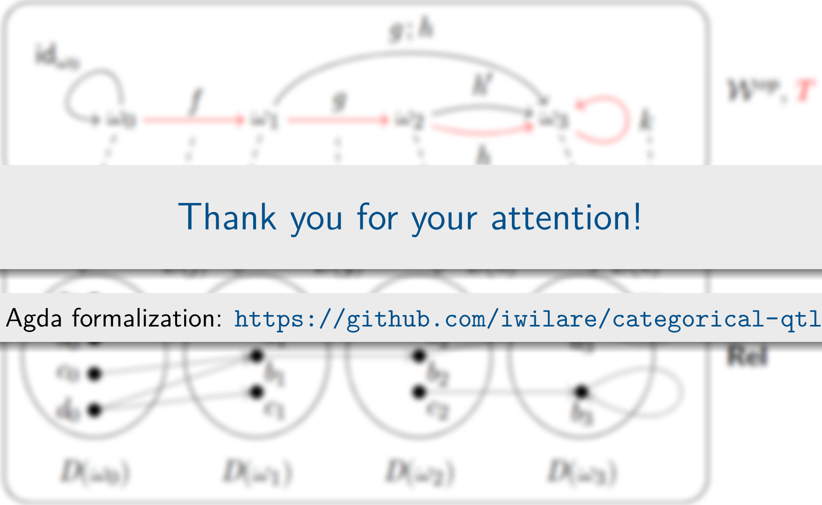

Thank you for your attention!

Agda formalization: `https://github.com/iwilare/categorical-qtl`