Specification and Verification of a Linear-Time Temporal Logic for Graph Transformation

Andrea Laretto[1], Fabio Gadducci[2], Davide Trotta[2]

1: Tallinn University of Technology, 2: University of Pisa

ICGT 2023, Leicester

July 19th, 2023

Contribution and Outline

We present the semantics of a counterpart-based temporal logic to reason on the evolution of graph-like structures, and formalize it using the proof assistant Agda along with results on its PNF.

Contribution and Outline

We present the semantics of a counterpart-based temporal logic to reason on the evolution of graph-like structures, and formalize it using the proof assistant Agda along with results on its PNF.

1. Temporal logics and counterpart paradigm

Contribution and Outline

We present the semantics of a counterpart-based temporal logic to reason on the evolution of graph-like structures, and formalize it using the proof assistant Agda along with results on its PNF.

1. Temporal logics and counterpart paradigm
2. Classical and categorical semantics

Contribution and Outline

We present the semantics of a counterpart-based temporal logic to reason on the evolution of graph-like structures, and formalize it using the proof assistant Agda along with results on its PNF.

1. Temporal logics and counterpart paradigm
2. Classical and categorical semantics
3. Examples and positive normal form

Contribution and Outline

We present the semantics of a counterpart-based temporal logic to reason on the evolution of graph-like structures, and formalize it using the proof assistant Agda along with results on its PNF.

1. Temporal logics and counterpart paradigm
2. Classical and categorical semantics
3. Examples and positive normal form
4. Agda formalization

Contribution and Outline

We present the semantics of a counterpart-based temporal logic to reason on the evolution of graph-like structures, and formalize it using the proof assistant Agda along with results on its PNF.

1. Temporal logics and counterpart paradigm
2. Classical and categorical semantics
3. Examples and positive normal form
4. Agda formalization
5. Conclusion and future work

Temporal logics

Well-known formalism for specifying and verifying complex systems

Temporal logics

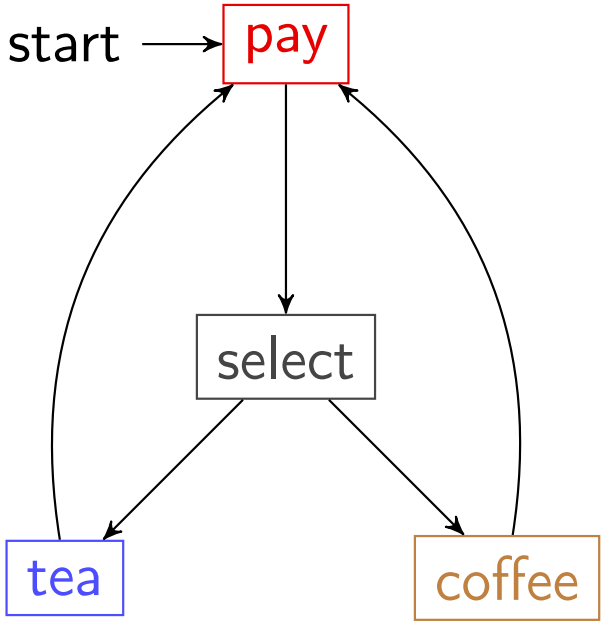
Well-known formalism for specifying and verifying complex systems

1. Represent the system as a **transition system**, called *model*

Temporal logics

Well-known formalism for specifying and verifying complex systems

1. Represent the system as a **transition system**, called *model*

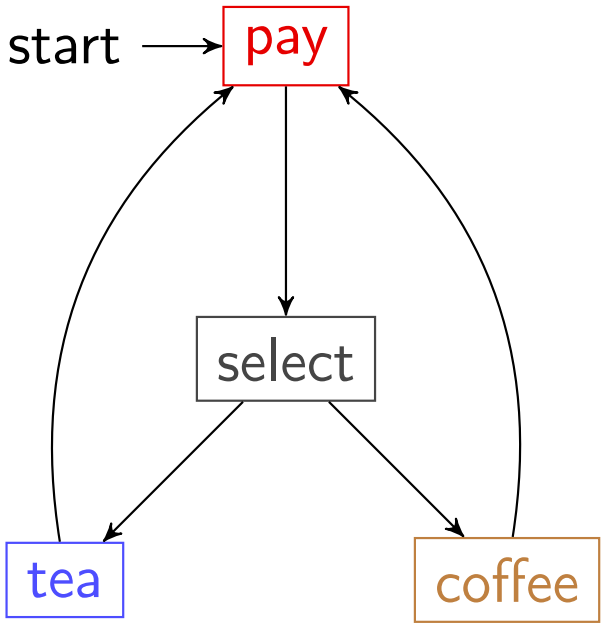


Transition system for a simple vending machine

Temporal logics

Well-known formalism for specifying and verifying complex systems

1. Represent the system as a **transition system**, called *model*



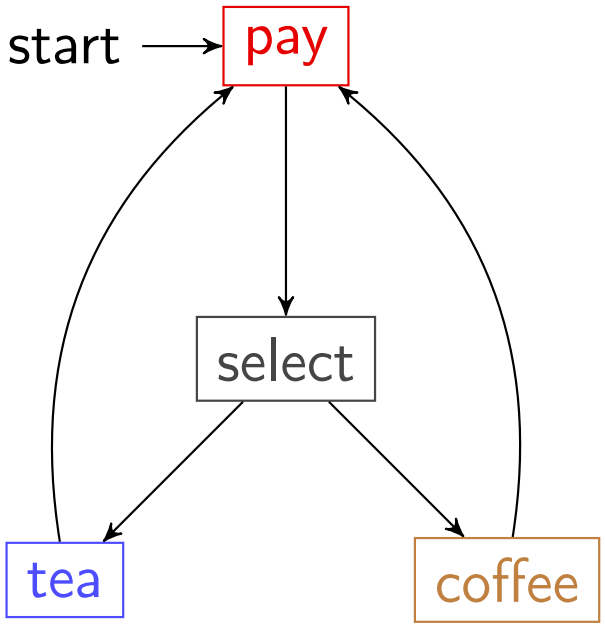
Transition system for a simple vending machine

2. Express desired properties as *formulas* in a **temporal logic**

Temporal logics

Well-known formalism for specifying and verifying complex systems

1. Represent the system as a **transition system**, called *model*



Transition system for a simple vending machine

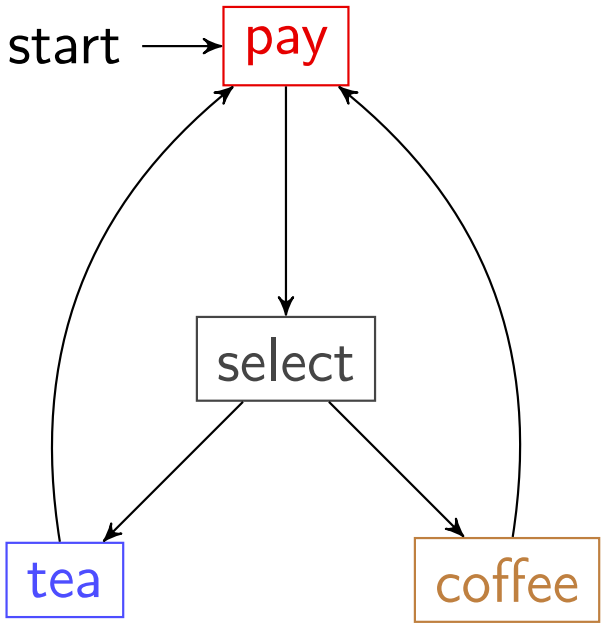
2. Express desired properties as *formulas* in a **temporal logic**

$$\mathrm{Always}(\mathrm{Eventually}(\mathrm{pay}))$$

Temporal logics

Well-known formalism for specifying and verifying complex systems

1. Represent the system as a **transition system**, called *model*



Transition system for a simple vending machine

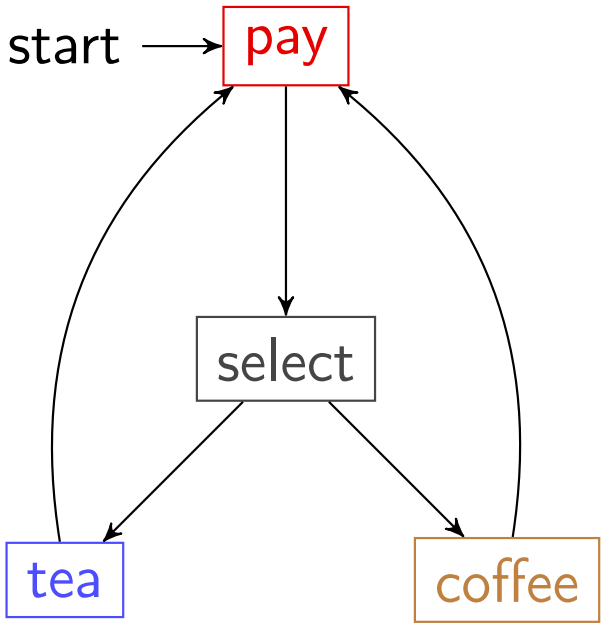
2. Express desired properties as *formulas* in a **temporal logic**

$$\mathrm{Always}(\mathrm{Eventually}(\mathrm{pay})) \qquad \neg\,\mathrm{Eventually}(\mathrm{tea})$$

Temporal logics

Well-known formalism for specifying and verifying complex systems

1. Represent the system as a **transition system**, called *model*



Transition system for a simple vending machine

2. Express desired properties as *formulas* in a **temporal logic**

$$\mathsf{Always}(\mathsf{Eventually}(\mathsf{pay})) \qquad \neg\,\mathsf{Eventually}(\mathsf{tea})$$

3. Use a program to *check* that the *model* **satisfies** the formula

Motivation: Multi-component models

- States are simply atomic points

Motivation: Multi-component models

- States are simply atomic points
- In practice, states often have structure that can change in time:

Motivation: Multi-component models

- States are simply atomic points
- In practice, states often have structure that can change in time:
  - Time evolution of *graph topologies*: merging nodes, deletion of edges

Motivation: Multi-component models

- States are simply atomic points
- In practice, states often have structure that can change in time:
  - Time evolution of *graph topologies*: merging nodes, deletion of edges
  - Managing *processes* in memory: forking, allocation and deallocation

Motivation: Multi-component models

- States are simply atomic points
- In practice, states often have structure that can change in time:
  - Time evolution of *graph topologies*: merging nodes, deletion of edges
  - Managing *processes* in memory: forking, allocation and deallocation
  - Dynamic behaviour of *election algorithms*: splitting and union of parties

Motivation: Multi-component models

- States are simply atomic points
- In practice, states often have structure that can change in time:
  - Time evolution of *graph topologies*: merging nodes, deletion of edges
  - Managing *processes* in memory: forking, allocation and deallocation
  - Dynamic behaviour of *election algorithms*: splitting and union of parties
- Objectives:

Can we enrich our models to express multi-component behaviour?

Motivation: Multi-component models

- States are simply atomic points
- In practice, states often have structure that can change in time:
  - Time evolution of *graph topologies*: merging nodes, deletion of edges
  - Managing *processes* in memory: forking, allocation and deallocation
  - Dynamic behaviour of *election algorithms*: splitting and union of parties
- Objectives:

Can we enrich our models to express multi-component behaviour?

Can we define logics that can reason on the fate of individual elements?

Motivation: Multi-component models

- States are simply atomic points
- In practice, states often have structure that can change in time:
  - Time evolution of *graph topologies*: merging nodes, deletion of edges
  - Managing *processes* in memory: forking, allocation and deallocation
  - Dynamic behaviour of *election algorithms*: splitting and union of parties
- Objectives:

  Can we enrich our models to express multi-component behaviour?
  Can we define logics that can reason on the fate of individual elements?

- *Yes!* Using **counterpart models** and quantified temporal logics

Counterpart paradigm

- Standard LTL traces: *sequences of states*

Counterpart paradigm

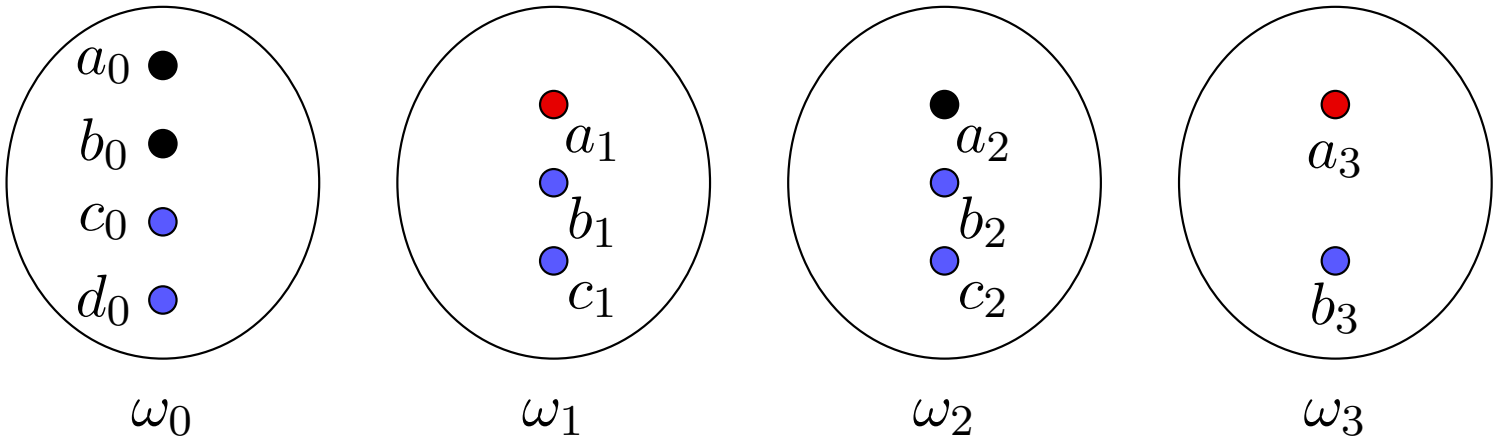
- Standard LTL traces: *sequences of states*

- Associate to each state a set of individuals, called **worlds**

Counterpart paradigm

- Standard LTL traces: *sequences of states*

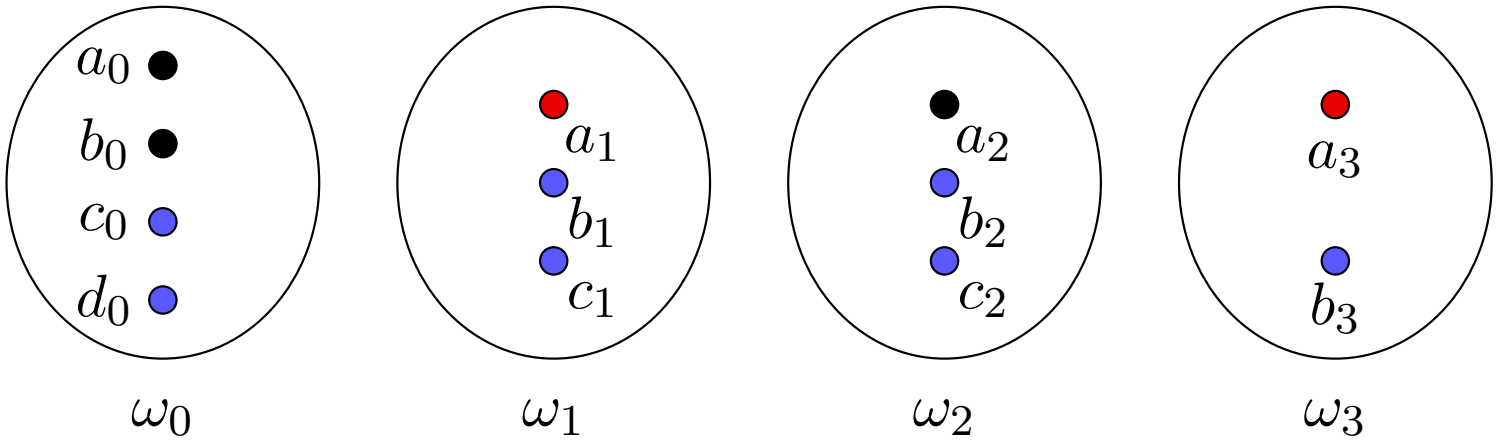
- Associate to each state a set of individuals, called **worlds**
- Our traces: *sequences of worlds*



Counterpart paradigm

- Standard LTL traces: *sequences of states*

- Associate to each state a set of individuals, called **worlds**
- Our traces: *sequences of worlds*

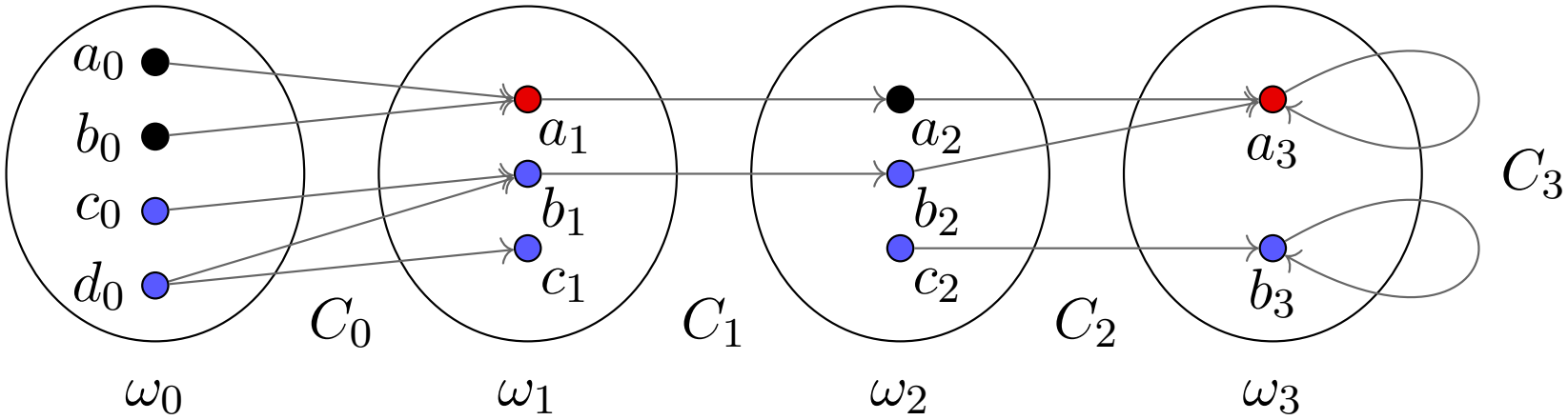


How do we represent transitions?

Counterpart paradigm

- Standard LTL traces: *sequences of states*

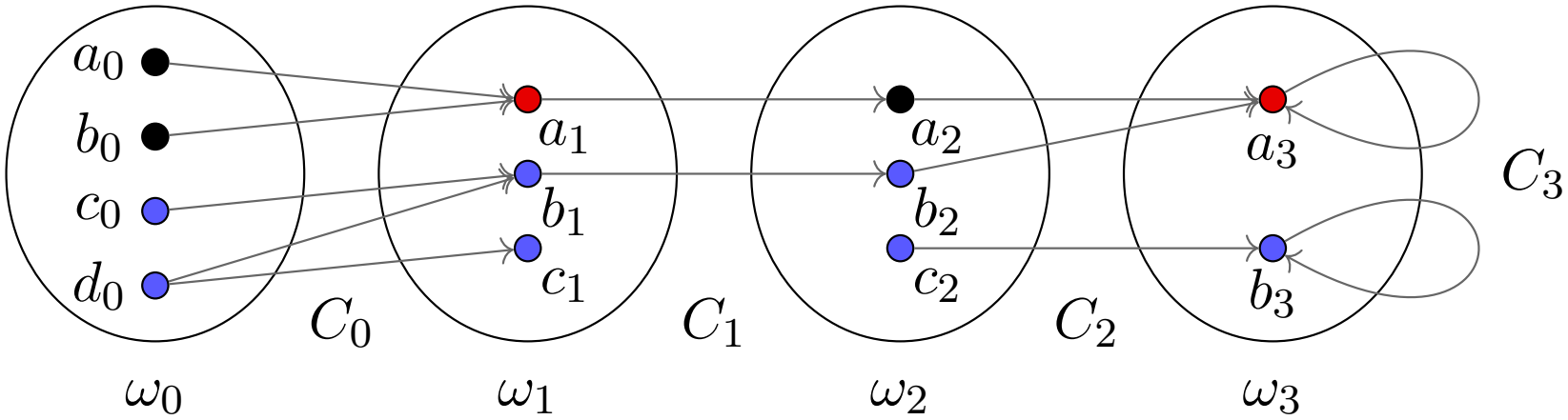
- Associate to each state a set of individuals, called **worlds**
- Our traces: *sequences of worlds, connected with* ***counterpart relations***



Counterpart paradigm

- Standard LTL traces: *sequences of states*

- Associate to each state a set of individuals, called **worlds**
- Our traces: *sequences of worlds, connected with* ***counterpart relations***

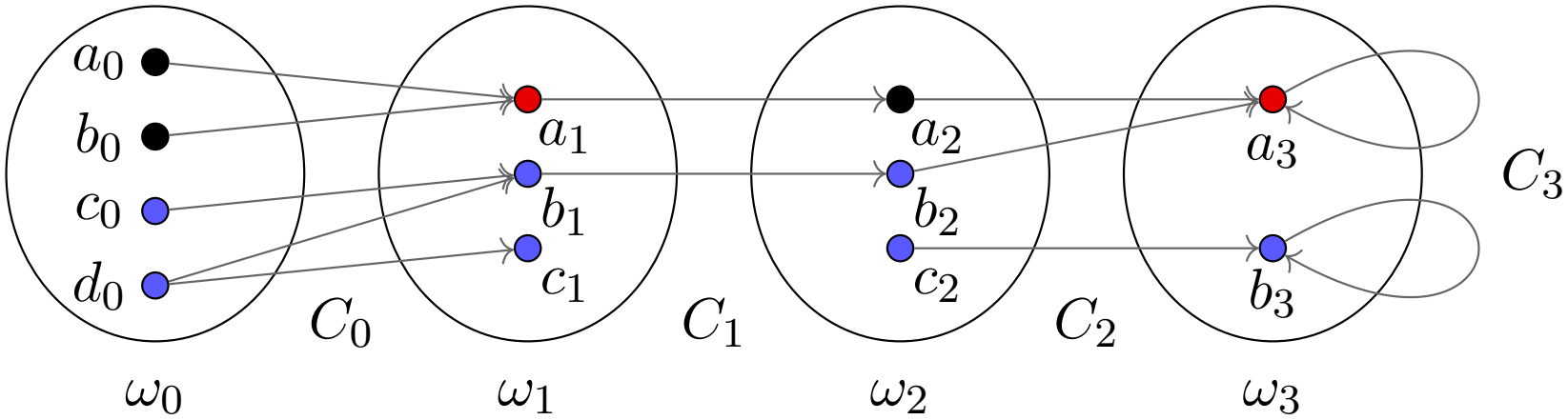


- Intuition: individuals connected by a relation *are the same after one step*

Counterpart paradigm

- Standard LTL traces: *sequences of states*

- Associate to each state a set of individuals, called **worlds**
- Our traces: *sequences of worlds, connected with* ***counterpart relations***



- Intuition: individuals connected by a relation *are the same after one step*
- We call these sequences of worlds and relations **counterpart traces**

Counterpart traces with algebras

- Counterpart trace: function $\omega : \mathbb{N} \to \mathbf{Set}$ and $\{R_i \subseteq \omega(i) \times \omega(i+1)\}_{i \in \mathbb{N}}$

Counterpart traces with algebras

- Counterpart trace: function $\omega : \mathbb{N} \to \textbf{Set}$ and $\{R_i \subseteq \omega(i) \times \omega(i+1)\}_{i \in \mathbb{N}}$
- *Worlds-as-algebras*: generalize sets to *algebras over a signature* Σ

Counterpart traces with algebras

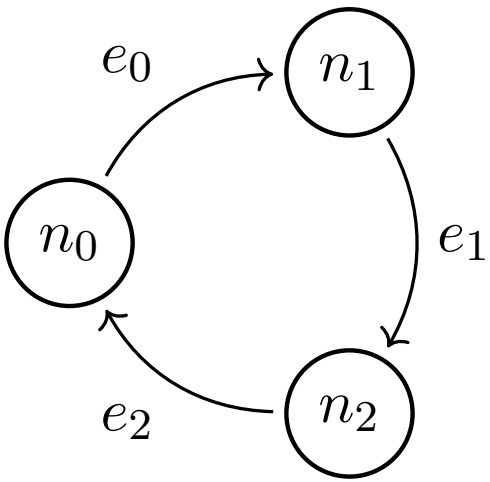
- Counterpart trace: function $\omega : \mathbb{N} \to \textbf{Set}$ and $\{R_i \subseteq \omega(i) \times \omega(i+1)\}_{i \in \mathbb{N}}$
- *Worlds-as-algebras*: generalize sets to *algebras over a signature* Σ
- *Idea:* take Σ-algebras and structure-preserving relations between them

Counterpart traces with algebras

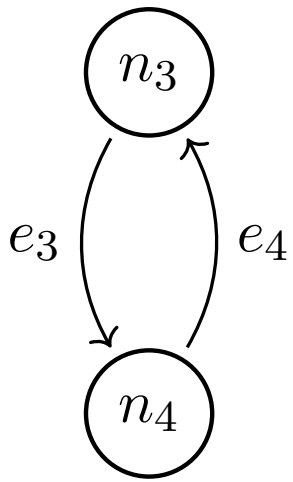
- Counterpart trace: function $\omega : \mathbb{N} \to \textbf{Set}$ and $\{R_i \subseteq \omega(i) \times \omega(i+1)\}_{i \in \mathbb{N}}$
- *Worlds-as-algebras*: generalize sets to *algebras over a signature* Σ
- *Idea:* take Σ-algebras and structure-preserving relations between them
- Examples: (multi)graphs, undirected graphs, trees, lists, etc.

Counterpart traces with algebras

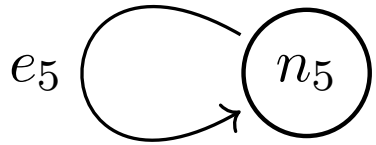
- Counterpart trace: function $\omega : \mathbb{N} \to \mathbf{Set}$ and $\{R_i \subseteq \omega(i) \times \omega(i+1)\}_{i \in \mathbb{N}}$
- *Worlds-as-algebras*: generalize sets to *algebras over a signature* Σ
- *Idea:* take Σ-algebras and structure-preserving relations between them
- Examples: (multi)graphs, undirected graphs, trees, lists, etc.
- A counterpart trace on the signature of directed graphs:



ω_0



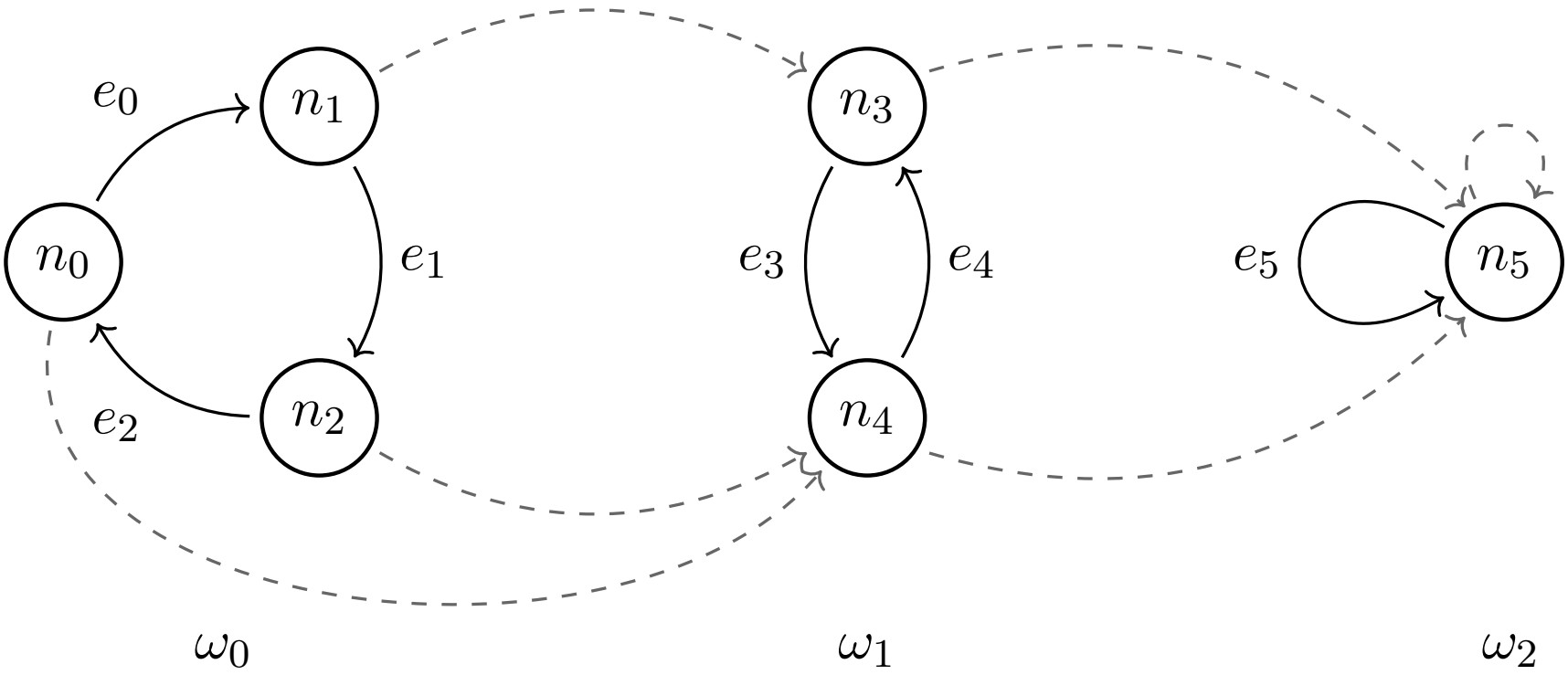
ω_1



ω_2

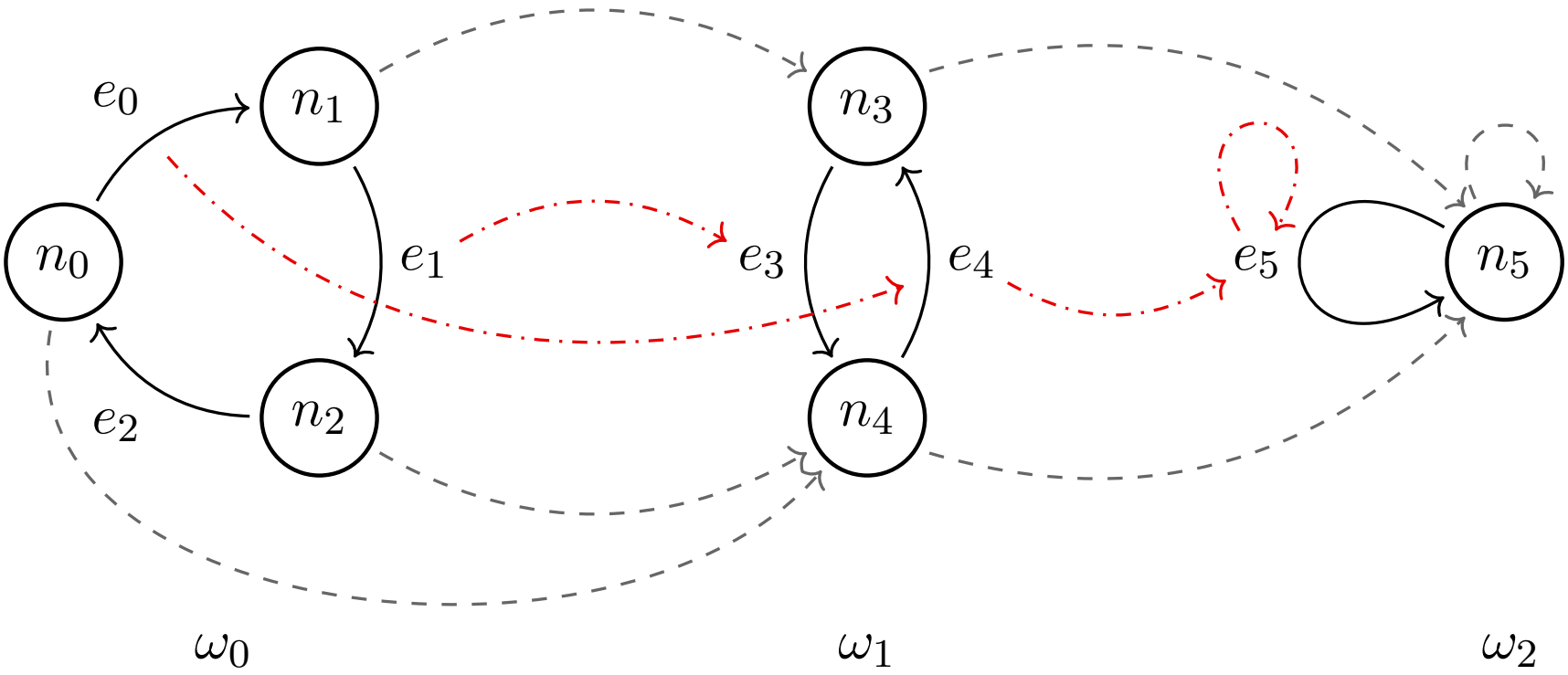
Counterpart traces with algebras

- Counterpart trace: function $\omega : \mathbb{N} \to \mathbf{Set}$ and $\{R_i \subseteq \omega(i) \times \omega(i+1)\}_{i \in \mathbb{N}}$
- *Worlds-as-algebras*: generalize sets to *algebras over a signature* Σ
- *Idea:* take Σ-algebras and structure-preserving relations between them
- Examples: (multi)graphs, undirected graphs, trees, lists, etc.
- A counterpart trace on the signature of directed graphs:



Counterpart traces with algebras

- Counterpart trace: function $\omega : \mathbb{N} \to \mathbf{Set}$ and $\{R_i \subseteq \omega(i) \times \omega(i+1)\}_{i \in \mathbb{N}}$
- *Worlds-as-algebras*: generalize sets to *algebras over a signature* Σ
- *Idea:* take Σ-algebras and structure-preserving relations between them
- Examples: (multi)graphs, undirected graphs, trees, lists, etc.
- A counterpart trace on the signature of directed graphs:



Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them

Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them
- Counterpart models can be understood within the unifying perspective of *category theory and categorical logic:*

Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them
- Counterpart models can be understood within the unifying perspective of *category theory and categorical logic:*

$$\textit{Counterpart model} \approx \textit{ a category } \mathcal{W}$$

Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them
- Counterpart models can be understood within the unifying perspective of *category theory and categorical logic:*

$$\textit{Counterpart model} \approx \textit{a category } \mathcal{W} + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}}$$

Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them
- Counterpart models can be understood within the unifying perspective of *category theory and categorical logic:*

$$\begin{aligned} \textit{Counterpart model} \approx\ & \textit{a category } \mathcal{W} \\ & + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}} \\ & + \underbrace{\textit{a presheaf } D : \mathcal{W}^{op} \to \textbf{\textit{Rel}}}_{\textit{Relational presheaf}} \end{aligned}$$

Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them
- Counterpart models can be understood within the unifying perspective of *category theory and categorical logic:*

$$\begin{aligned} \textit{Counterpart model} \approx\ & \textit{a category } \mathcal{W} \\ & + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}} \\ & + \underbrace{\textit{a presheaf } D : \mathcal{W}^{op} \to \textbf{\textit{Rel}}}_{\textit{Relational presheaf}} \end{aligned}$$

- Objects of $\mathcal{W}$ are the states of the underlying *transition system*

Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them
- Counterpart models can be understood within the unifying perspective of *category theory and categorical logic:*

$$\begin{aligned} \textit{Counterpart model} \approx\ & \textit{a category } \mathcal{W} \\ & + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}} \\ & + \underbrace{\textit{a presheaf } D : \mathcal{W}^{op} \to \textbf{\textit{Rel}}}_{\textit{Relational presheaf}} \end{aligned}$$

- Objects of $\mathcal{W}$ are the states of the underlying *transition system*
- Morphisms of $\mathcal{W}$ represent *transitions* between states

Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them
- Counterpart models can be understood within the unifying perspective of *category theory and categorical logic:*

$$\begin{aligned} \textit{Counterpart model} \approx\ & \textit{a category } \mathcal{W} \\ & + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}} \\ & + \underbrace{\textit{a presheaf } D : \mathcal{W}^{op} \to \textbf{\textit{Rel}}}_{\textit{Relational presheaf}} \end{aligned}$$

- Objects of $\mathcal{W}$ are the states of the underlying *transition system*
- Morphisms of $\mathcal{W}$ represent *transitions* between states
- The *temporal structure* identifies the one-step transitions of the model

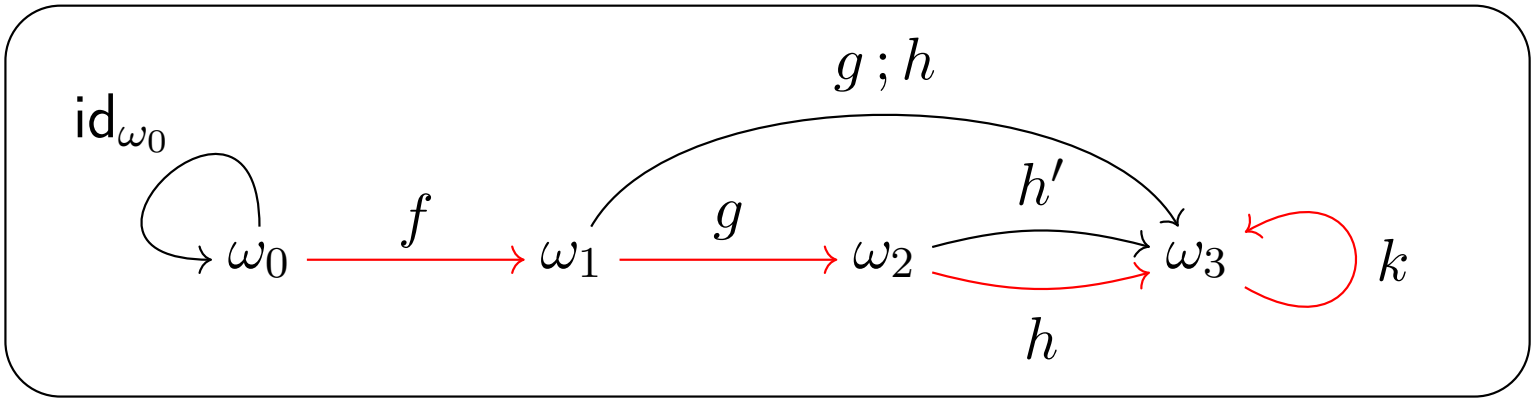
Categorical semantics

- *Counterpart model*: a transition system enriched with worlds and counterpart relations between them
- Counterpart models can be understood within the unifying perspective of *category theory and categorical logic:*

$$\begin{aligned} \textit{Counterpart model} \approx\ & \textit{a category } \mathcal{W} \\ & + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}} \\ & + \underbrace{\textit{a presheaf } D : \mathcal{W}^{op} \to \textbf{\textit{Rel}}}_{\textit{Relational presheaf}} \end{aligned}$$

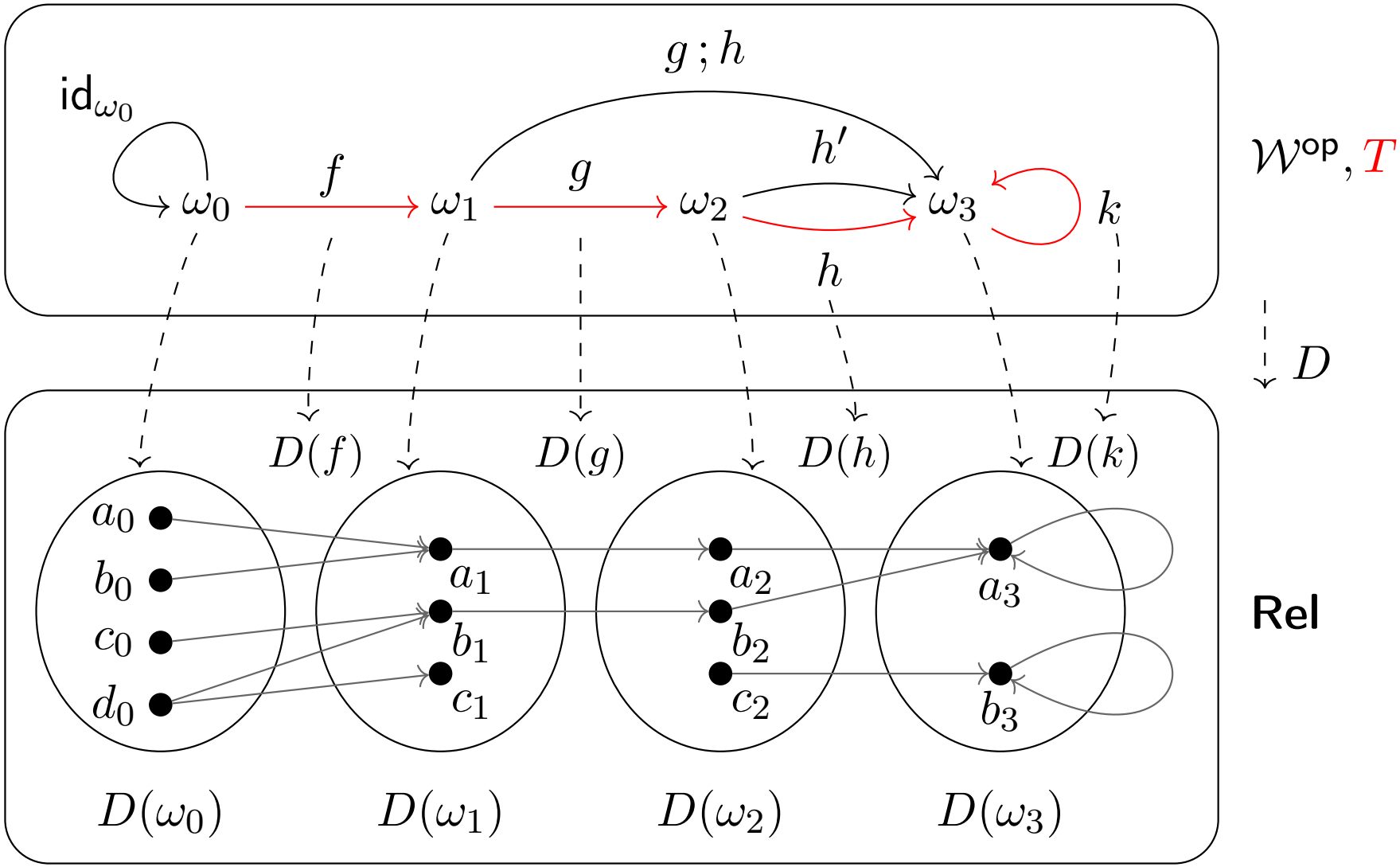
- Objects of $\mathcal{W}$ are the states of the underlying *transition system*
- Morphisms of $\mathcal{W}$ represent *transitions* between states
- The *temporal structure* identifies the one-step transitions of the model
- The *relational presheaf* assign worlds and counterpart relations to states

Example – Counterpart model



$\mathcal{W}^{\mathsf{op}}, T$

Example – Counterpart model



Categorical semantics

- For the signature of directed graphs:

$$\textit{Graph counterpart model} \approx \textit{a category } \mathcal{W} + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}}$$

Categorical semantics

- For the signature of directed graphs:

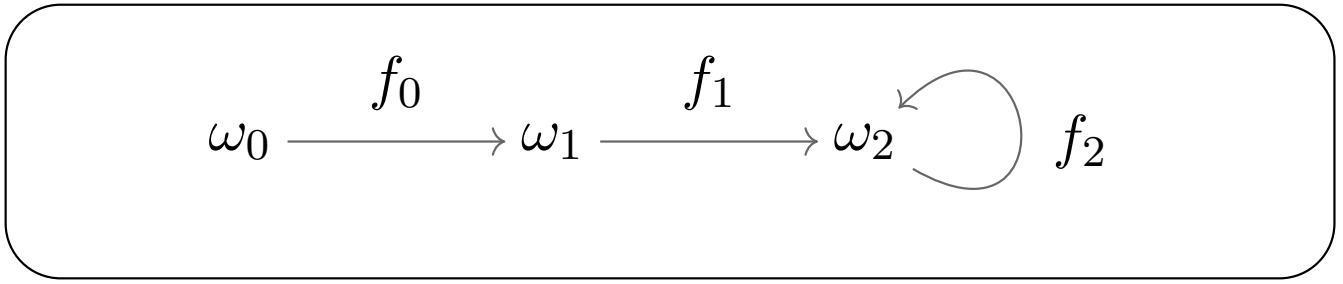
$$\begin{aligned} \textit{Graph counterpart model} \approx\ & \textit{a category } \mathcal{W} \\ & + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}} \\ & + \underbrace{\textit{relational presheaves } N, E : \mathcal{W}^{op} \to \textbf{\textit{Rel}}}_{\textit{Sorts of the signature}} \end{aligned}$$

Categorical semantics

- For the signature of directed graphs:

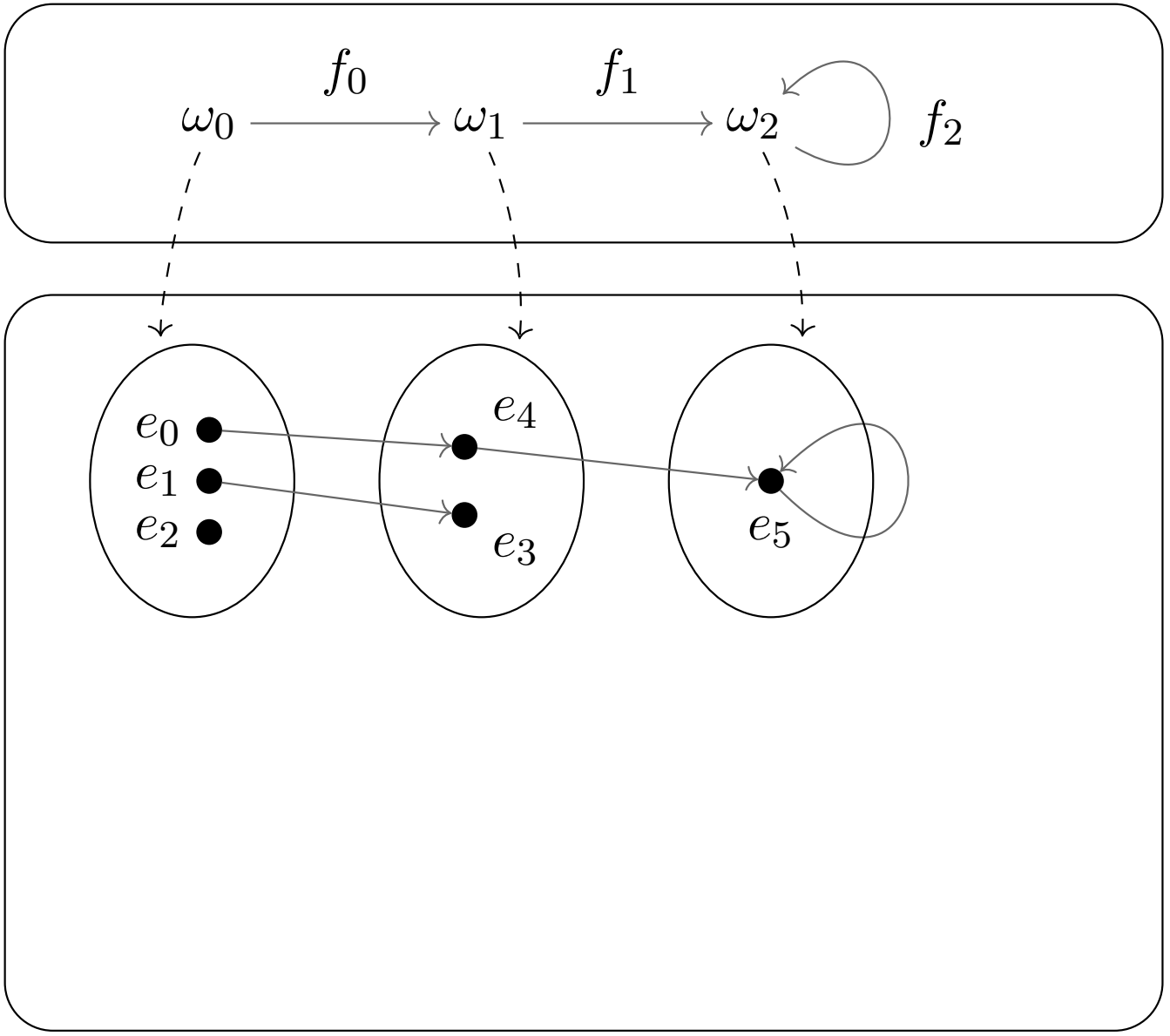
$$\begin{aligned} \textit{Graph counterpart model} \approx\ & \textit{a category } \mathcal{W} \\ & + \underbrace{\textit{a class } T \textit{ of selected morphisms of } \mathcal{W}}_{\textit{Temporal structure}} \\ & + \underbrace{\textit{relational presheaves } N, E : \mathcal{W}^{op} \to \textbf{\textit{Rel}}}_{\textit{Sorts of the signature}} \\ & + \underbrace{\textit{relational morphisms } s, t : E \Rightarrow N}_{\textit{Function symbols}} \end{aligned}$$

Example – Graph counterpart model



$\mathcal{W}^{\mathsf{op}}, T$

Example – Graph counterpart model

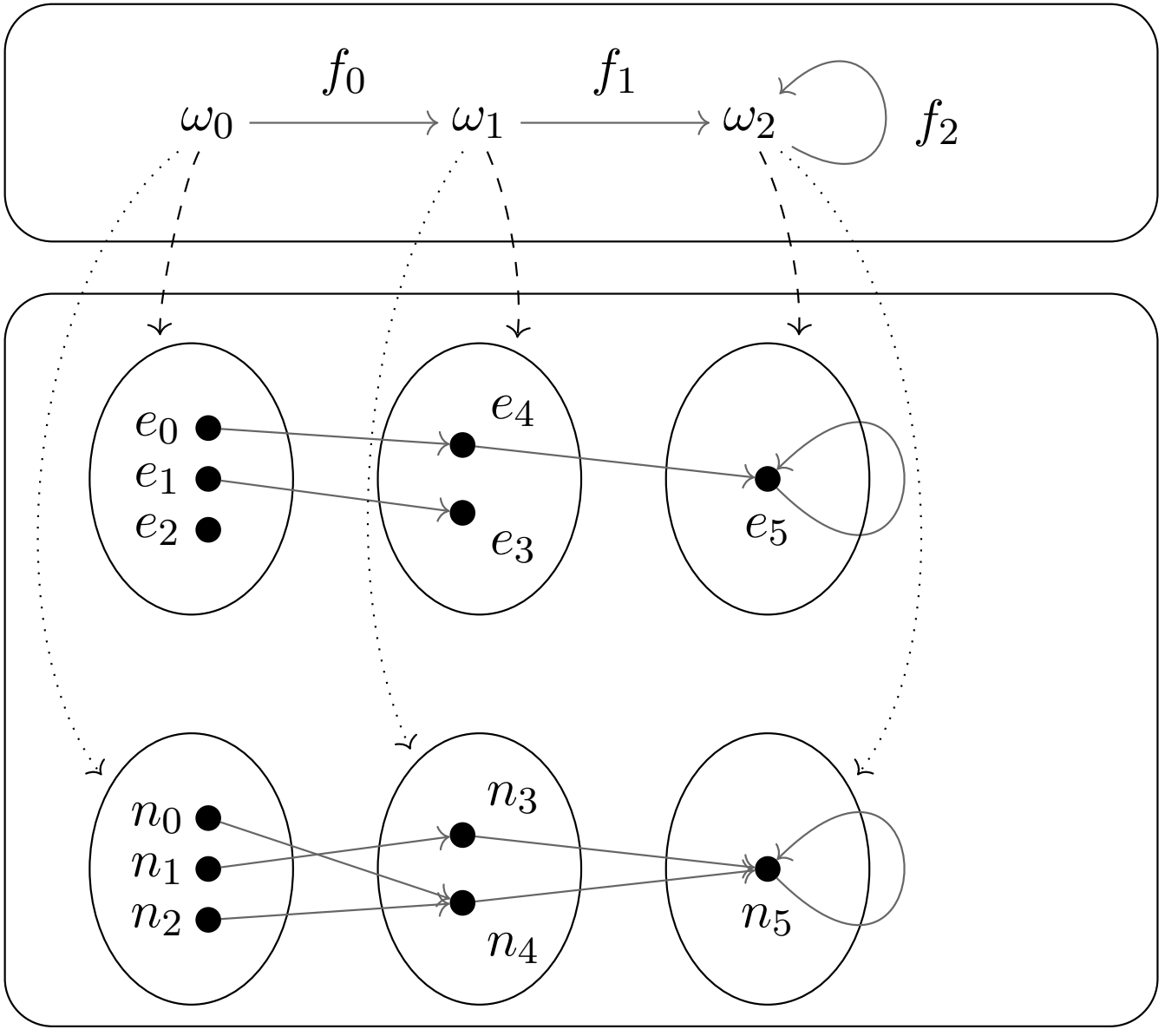


$\mathcal{W}^{\mathsf{op}}, T$

$[\![\mathsf{Edge}]\!]$

Rel

Example – Graph counterpart model

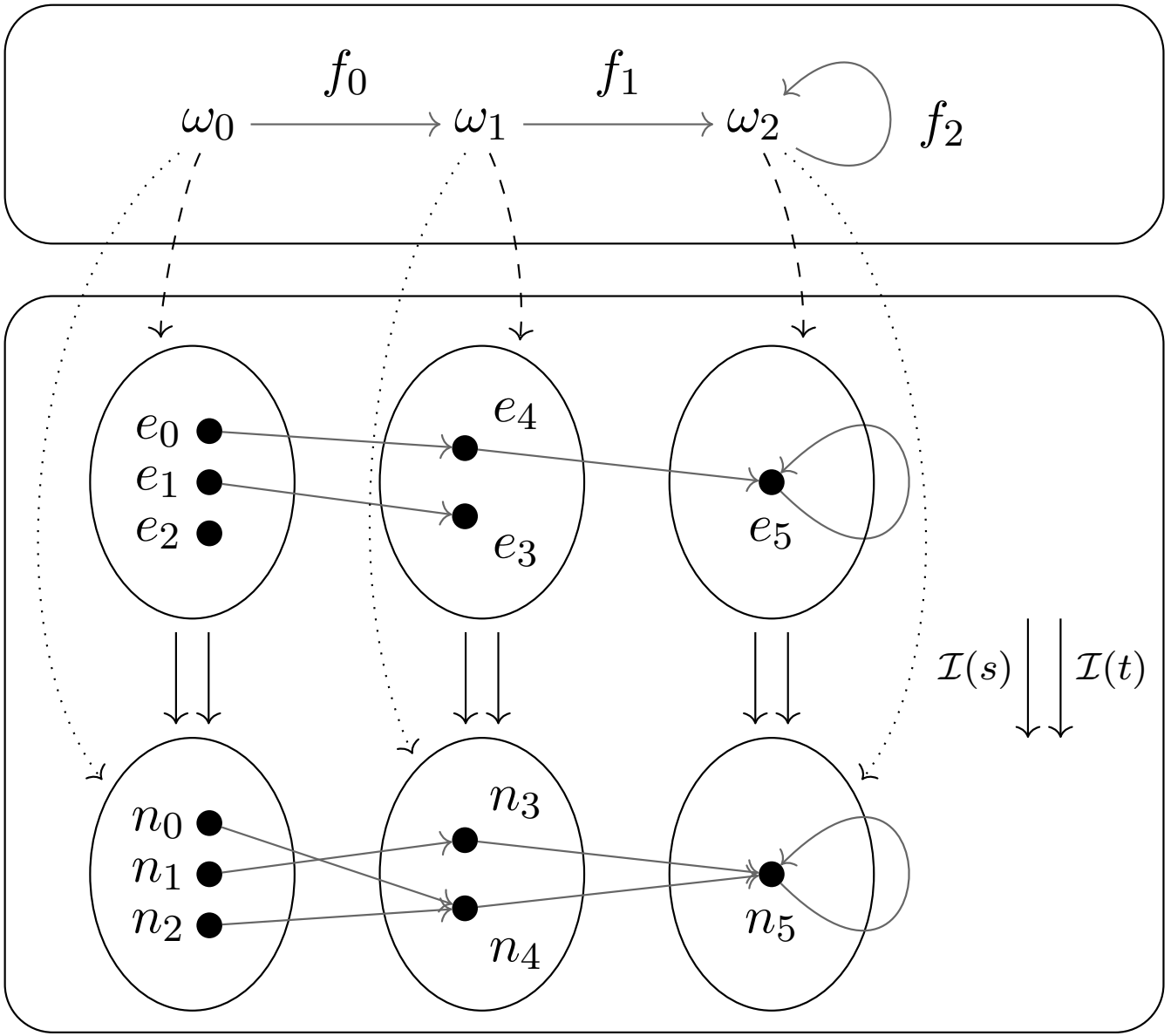


$\mathcal{W}^{\mathsf{op}}, T$

$[\![\text{Edge}]\!]$ $[\![\text{Node}]\!]$

Rel

Example – Graph counterpart model



$\mathcal{W}^{\mathsf{op}}, T$

$[\![\text{Edge}]\!]$ $[\![\text{Node}]\!]$

Rel

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \text{true} \mid \neg\phi \mid \phi \wedge \phi$$

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi$$

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x)$$

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi\,\mathsf{Until}\,\phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.
  - $\sigma, \mu \vDash \mathsf{true}$;

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.
  - $\sigma, \mu \vDash \mathsf{true}$;
  - $\sigma, \mu \vDash \phi_1 \wedge \phi_2$ iff $\sigma, \mu \vDash \phi_1$ and $\sigma, \mu \vDash \phi_2$;

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.
  - $\sigma, \mu \vDash \mathsf{true}$;
  - $\sigma, \mu \vDash \phi_1 \wedge \phi_2$ iff $\sigma, \mu \vDash \phi_1$ and $\sigma, \mu \vDash \phi_2$;
  - $\sigma, \mu \vDash e_1 =_{\mathsf{E}} e_2$ iff $\mu^*_{\mathsf{E}}(e_1) = \mu^*_{\mathsf{E}}(e_2)$;

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.
  - $\sigma, \mu \vDash \mathsf{true}$;
  - $\sigma, \mu \vDash \phi_1 \wedge \phi_2$ iff $\sigma, \mu \vDash \phi_1$ and $\sigma, \mu \vDash \phi_2$;
  - $\sigma, \mu \vDash e_1 =_{\mathsf{E}} e_2$ iff $\mu^*_{\mathsf{E}}(e_1) = \mu^*_{\mathsf{E}}(e_2)$;
  - $\sigma, \mu \vDash \exists_{\mathsf{N}} x.\phi$ iff there is a node $n \in D(\omega_0)_N$ such that $\sigma, \mu[x \mapsto n] \vDash \phi$;

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.
  - $\sigma, \mu \vDash \mathsf{true}$;
  - $\sigma, \mu \vDash \phi_1 \wedge \phi_2$ iff $\sigma, \mu \vDash \phi_1$ and $\sigma, \mu \vDash \phi_2$;
  - $\sigma, \mu \vDash e_1 =_{\mathsf{E}} e_2$ iff $\mu^*_{\mathsf{E}}(e_1) = \mu^*_{\mathsf{E}}(e_2)$;
  - $\sigma, \mu \vDash \exists_{\mathsf{N}} x.\phi$ iff there is a node $n \in D(\omega_0)_N$ such that $\sigma, \mu[x \mapsto n] \vDash \phi$;
  - $\sigma, \mu \vDash \mathsf{O}\phi$ iff there is an assignment μ_1 s.t. $\langle \mu, \mu_1 \rangle \in C_0$ and $\sigma_1, \mu_1 \vDash \phi$;

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.
  - $\sigma, \mu \vDash \mathsf{true}$;
  - $\sigma, \mu \vDash \phi_1 \wedge \phi_2$ iff $\sigma, \mu \vDash \phi_1$ and $\sigma, \mu \vDash \phi_2$;
  - $\sigma, \mu \vDash e_1 =_{\mathsf{E}} e_2$ iff $\mu^*_{\mathsf{E}}(e_1) = \mu^*_{\mathsf{E}}(e_2)$;
  - $\sigma, \mu \vDash \exists_{\mathsf{N}} x.\phi$ iff there is a node $n \in D(\omega_0)_N$ such that $\sigma, \mu[x \mapsto n] \vDash \phi$;
  - $\sigma, \mu \vDash \mathsf{O}\phi$ iff there is an assignment μ_1 s.t. $\langle \mu, \mu_1 \rangle \in C_0$ and $\sigma_1, \mu_1 \vDash \phi$;
  - $\sigma, \mu \vDash \phi_1 \mathsf{U} \phi_2$ iff there is an $\bar{n} \geq 0$ such that

QLTL

- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.
  - $\sigma, \mu \vDash \mathsf{true}$;
  - $\sigma, \mu \vDash \phi_1 \wedge \phi_2$ iff $\sigma, \mu \vDash \phi_1$ and $\sigma, \mu \vDash \phi_2$;
  - $\sigma, \mu \vDash e_1 =_{\mathsf{E}} e_2$ iff $\mu^*_{\mathsf{E}}(e_1) = \mu^*_{\mathsf{E}}(e_2)$;
  - $\sigma, \mu \vDash \exists_{\mathsf{N}} x.\phi$ iff there is a node $n \in D(\omega_0)_N$ such that $\sigma, \mu[x \mapsto n] \vDash \phi$;
  - $\sigma, \mu \vDash \mathsf{O}\phi$ iff there is an assignment μ_1 s.t. $\langle \mu, \mu_1 \rangle \in C_0$ and $\sigma_1, \mu_1 \vDash \phi$;
  - $\sigma, \mu \vDash \phi_1 \mathsf{U} \phi_2$ iff there is an $\bar{n} \geq 0$ such that
    1. for any $i < \bar{n}$, there is a μ_i such that $\langle \mu, \mu_i \rangle \in \sigma_{\leq i}$ and $\sigma_i, \mu_i \vDash \phi_1$;

QLTL

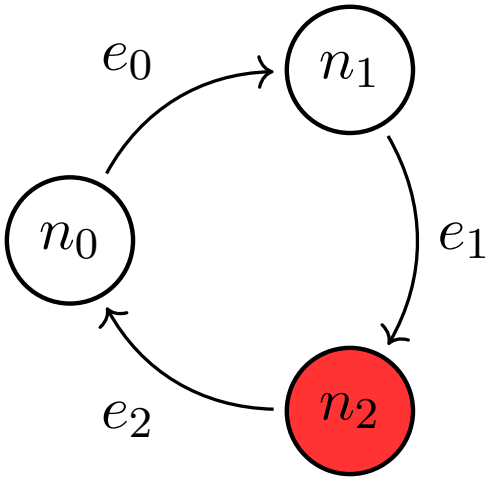
- QLTL: *(first-order) quantified linear temporal logic* using traces
- Syntax of QLTL formulas:

$$\phi := \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \mathsf{Next}(\phi) \mid \phi \,\mathsf{Until}\, \phi \mid \exists_{\mathsf{N}} x.\phi \mid \exists_{\mathsf{E}} x.\phi \mid P(x) \mid \psi$$

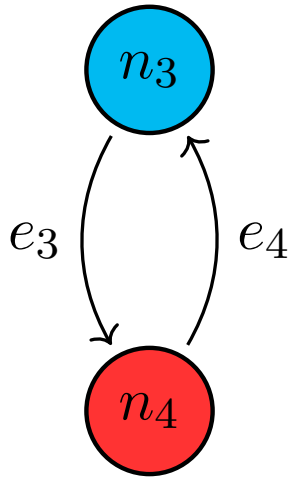
$$\psi := n =_{\mathsf{N}} n \mid e =_{\mathsf{E}} e, \quad \text{with} \quad n := x \mid s(e) \mid t(e), \quad \text{and} \quad e := x.$$

- Semantics: *given a trace* σ, define a satisfiability relation on (tuples of) nodes and edges satisfying ϕ, i.e., assignments μ for the $\mathrm{fv}(\phi)$.
  - $\sigma, \mu \vDash \mathsf{true}$;
  - $\sigma, \mu \vDash \phi_1 \wedge \phi_2$ iff $\sigma, \mu \vDash \phi_1$ and $\sigma, \mu \vDash \phi_2$;
  - $\sigma, \mu \vDash e_1 =_{\mathsf{E}} e_2$ iff $\mu^*_{\mathsf{E}}(e_1) = \mu^*_{\mathsf{E}}(e_2)$;
  - $\sigma, \mu \vDash \exists_{\mathsf{N}} x.\phi$ iff there is a node $n \in D(\omega_0)_N$ such that $\sigma, \mu[x \mapsto n] \vDash \phi$;
  - $\sigma, \mu \vDash \mathsf{O}\phi$ iff there is an assignment μ_1 s.t. $\langle \mu, \mu_1 \rangle \in C_0$ and $\sigma_1, \mu_1 \vDash \phi$;
  - $\sigma, \mu \vDash \phi_1 \mathsf{U} \phi_2$ iff there is an $\bar{n} \geq 0$ such that
    1. for any $i < \bar{n}$, there is a μ_i such that $\langle \mu, \mu_i \rangle \in \sigma_{\leq i}$ and $\sigma_i, \mu_i \vDash \phi_1$;
    2. there is a $\mu_{\bar{n}}$ such that $\langle \mu, \mu_{\bar{n}} \rangle \in \sigma_{\leq \bar{n}}$ and $\sigma_{\bar{n}}, \mu_{\bar{n}} \vDash \phi_2$;

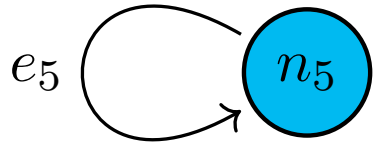
Example – QLTL



ω_0

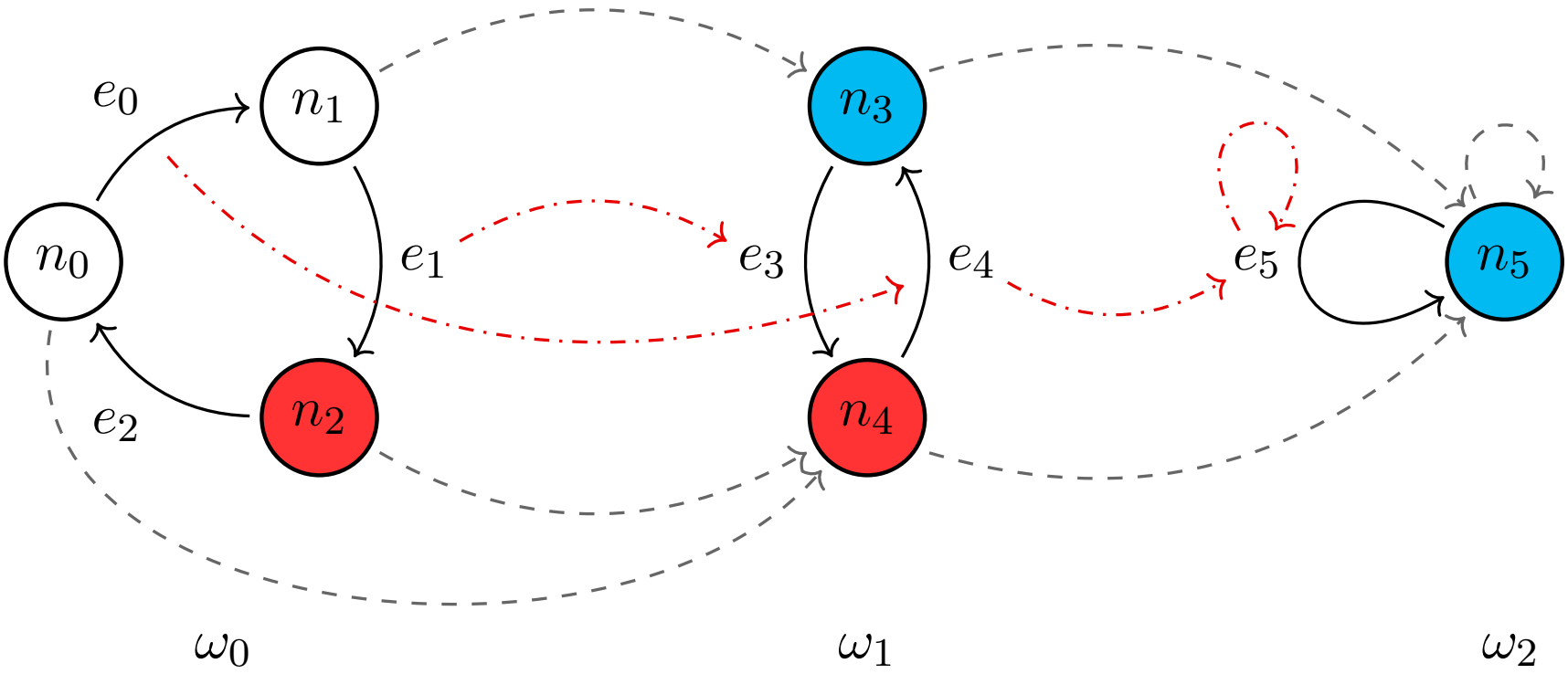


ω_1

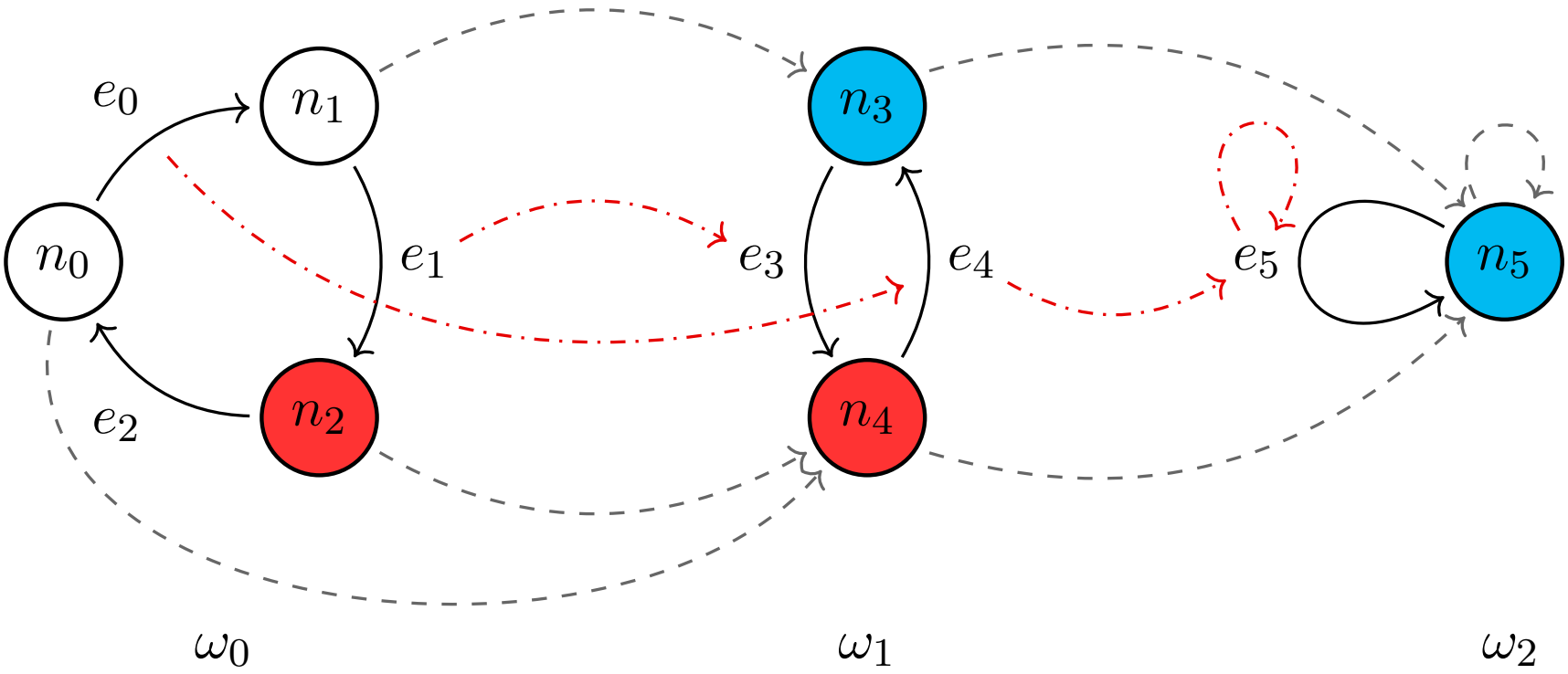


ω_2

Example – QLTL

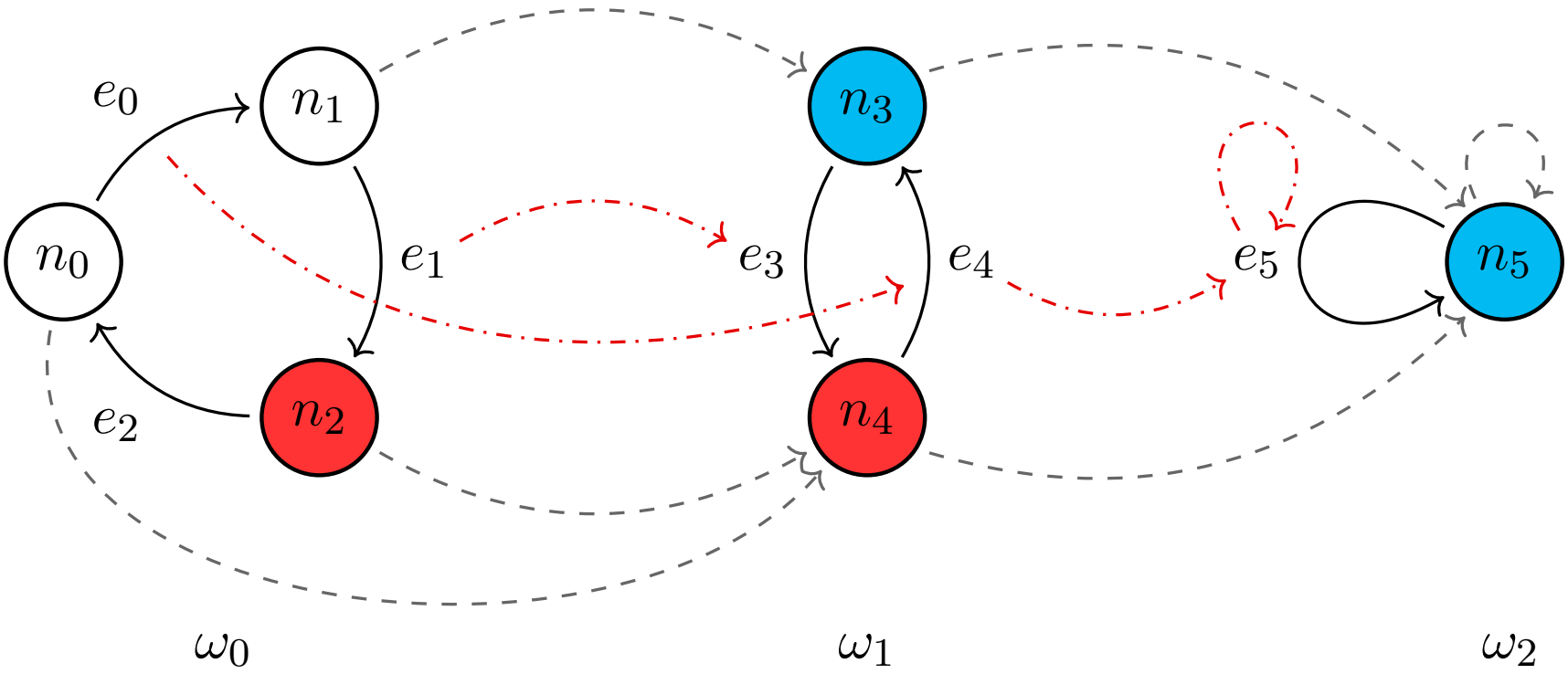


Example – QLTL



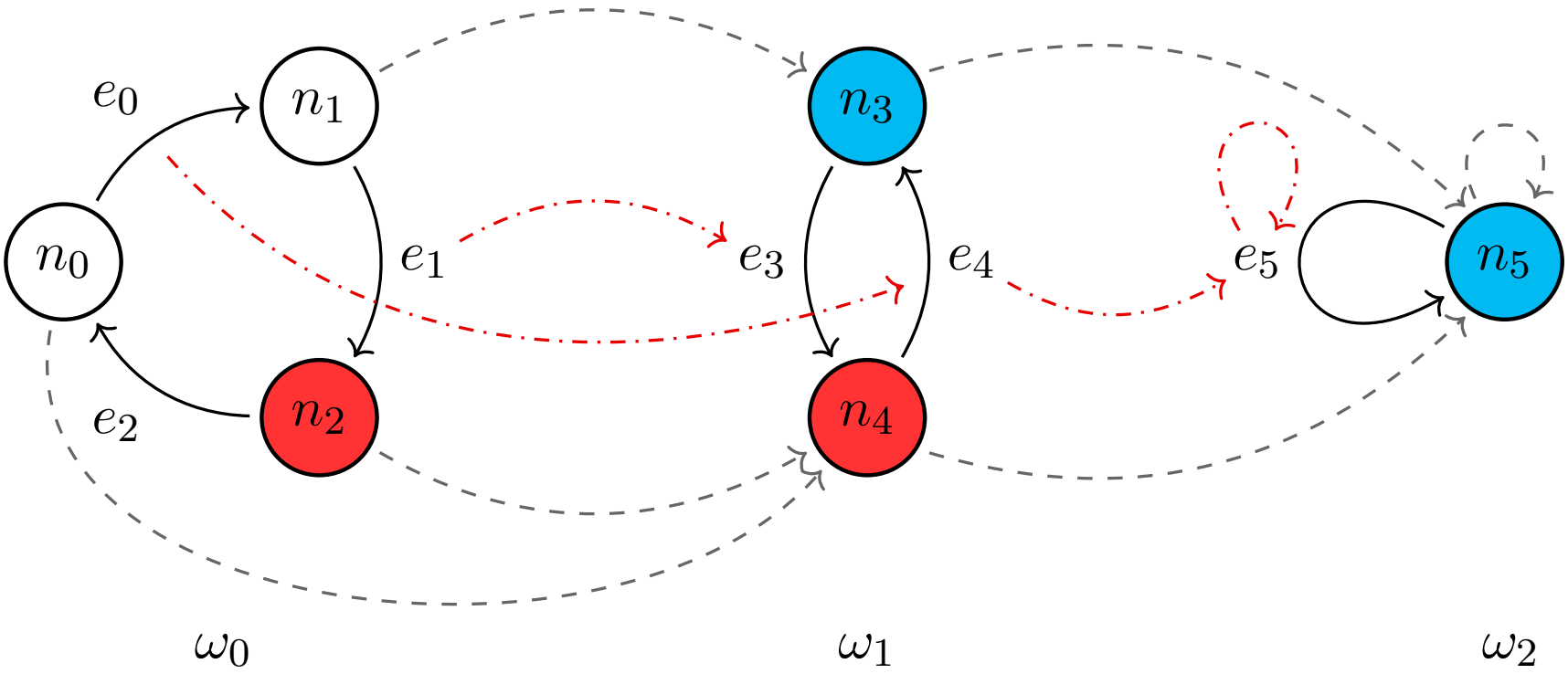
- $n_1 \vDash_{\omega_0} \mathsf{Next}(\mathsf{Blue}(x))$

Example – QLTL



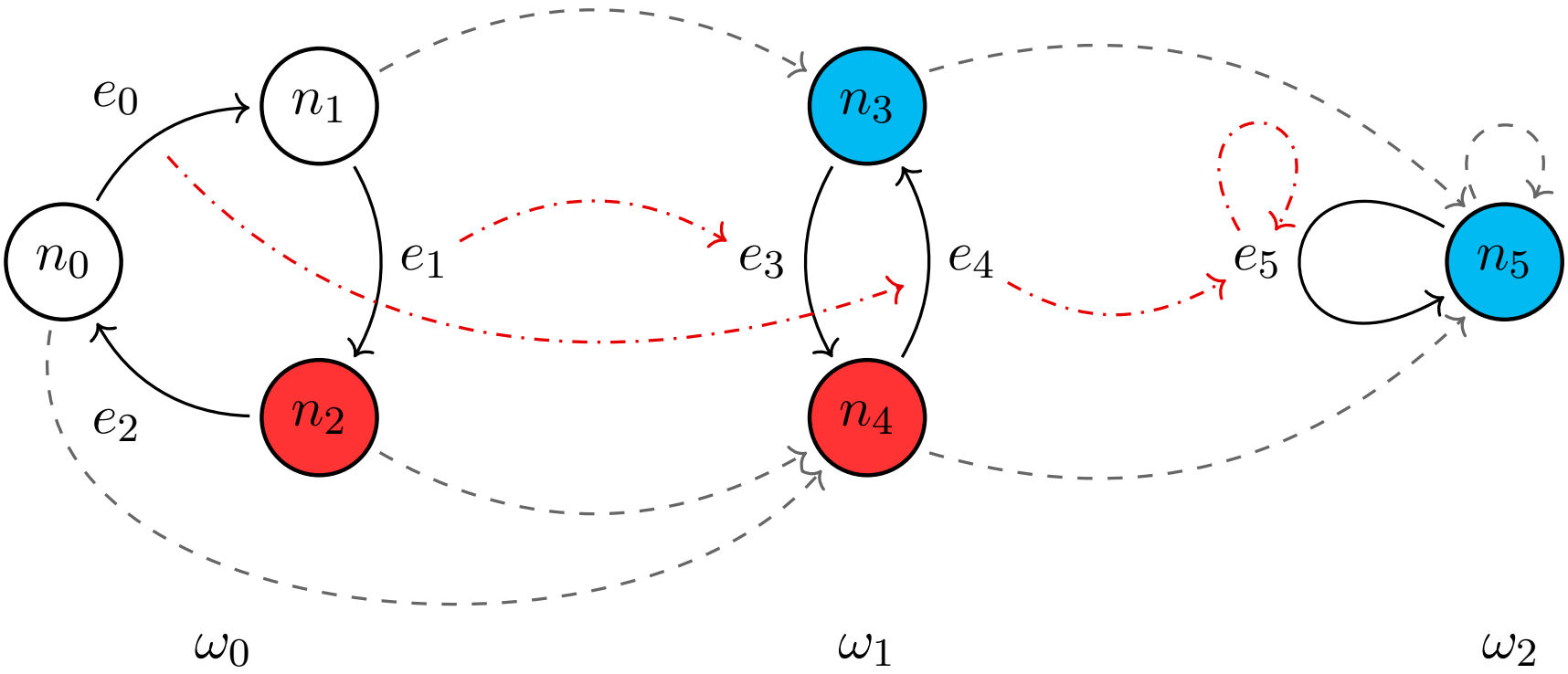
- $n_1 \models_{\omega_0} \mathsf{Next}(\mathsf{Blue}(x))$
- $n_0 \models_{\omega_0} \neg\mathsf{Next}(\mathsf{Red}(x))$

Example – QLTL



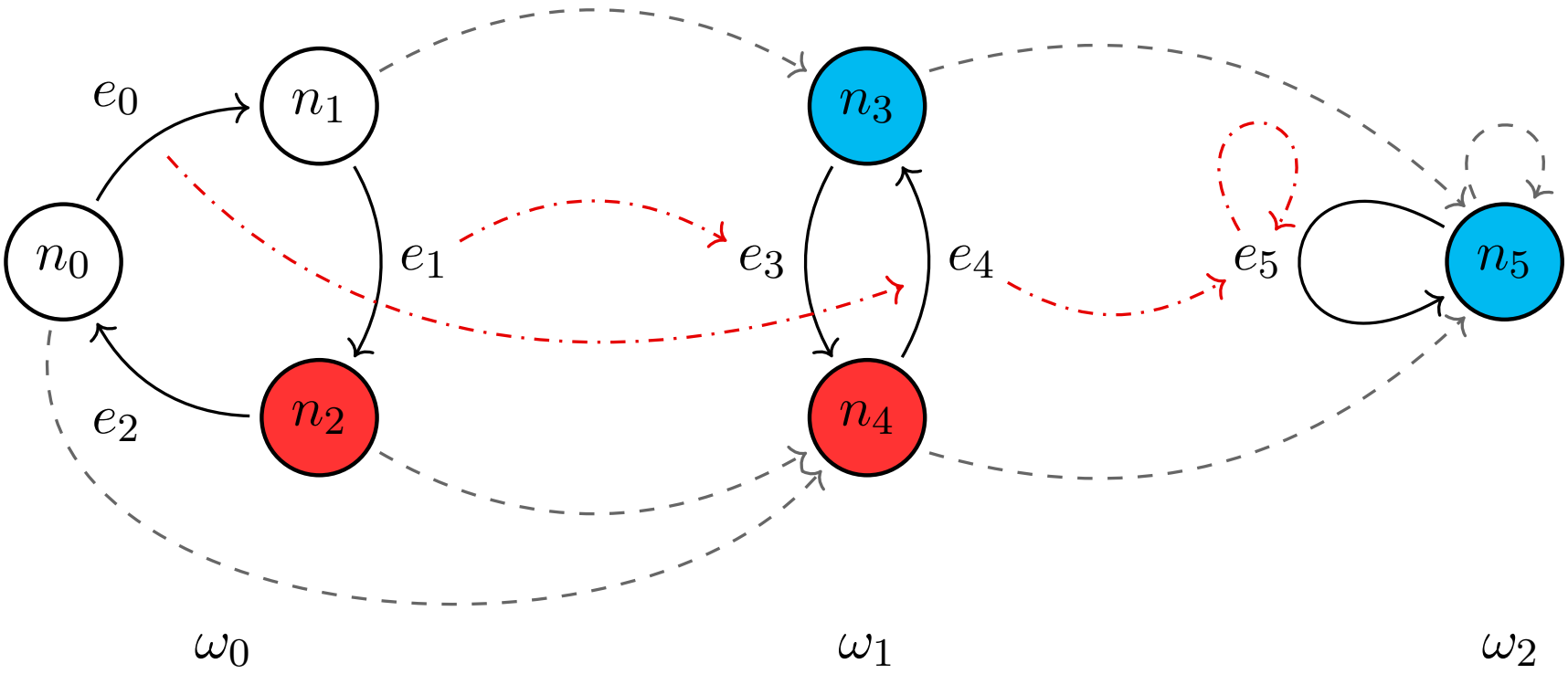
- $n_1 \models_{\omega_0} \mathsf{Next}(\mathsf{Blue}(x))$
- $n_0 \models_{\omega_0} \neg\mathsf{Next}(\mathsf{Red}(x))$
- $n_2 \models_{\omega_0} \mathsf{Red}(x)\ \mathsf{Until}\ \mathsf{Blue}(x)$

Example – QLTL



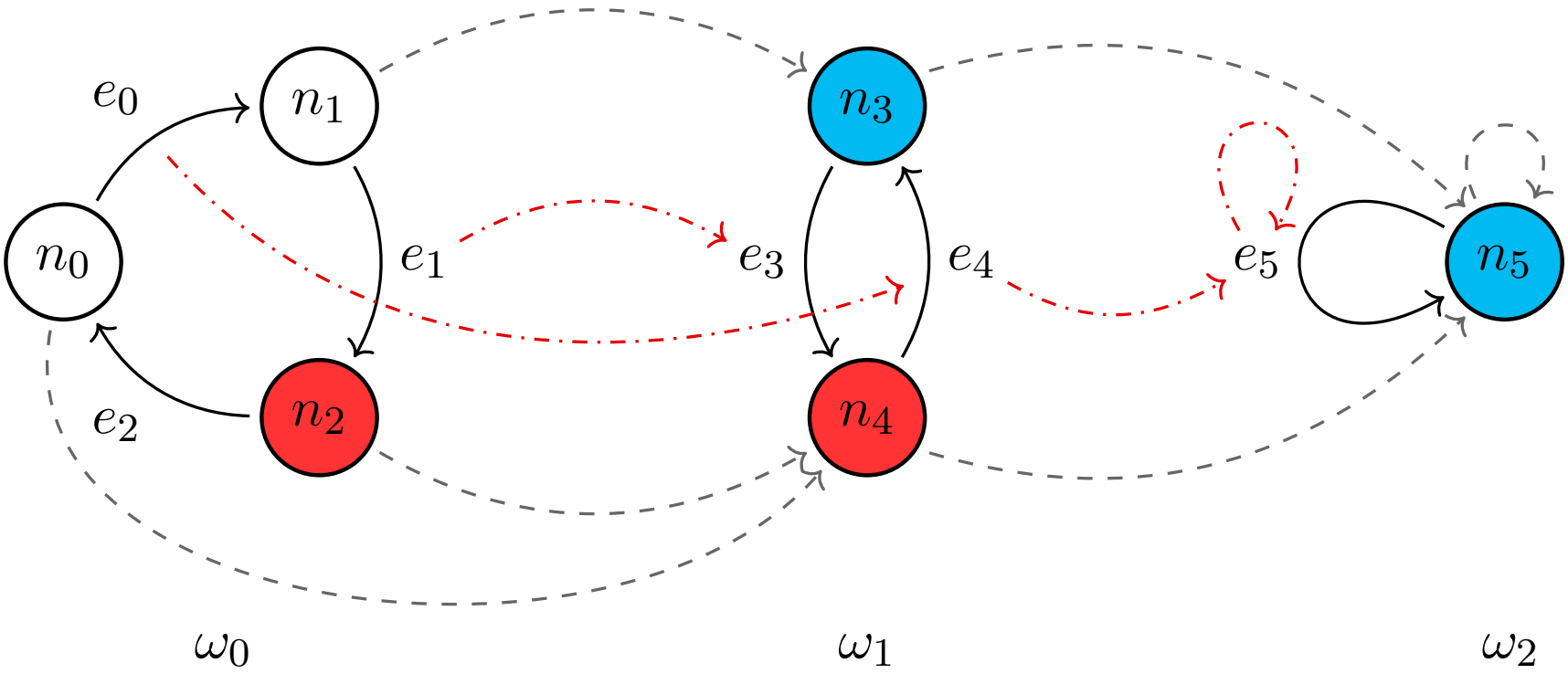
- $n_1 \vDash_{\omega_0} \mathsf{Next}(\mathsf{Blue}(x))$
- $n_0 \vDash_{\omega_0} \neg\mathsf{Next}(\mathsf{Red}(x))$
- $n_2 \vDash_{\omega_0} \mathsf{Red}(x)\ \mathsf{Until}\ \mathsf{Blue}(x)$
- $(n_3, n_4) \vDash_{\omega_1} \mathsf{Next}(x = y)$

Example – QLTL



- $n_1 \vDash_{\omega_0} \mathsf{Next}(\mathsf{Blue}(x))$
- $n_0 \vDash_{\omega_0} \neg\mathsf{Next}(\mathsf{Red}(x))$
- $n_2 \vDash_{\omega_0} \mathsf{Red}(x)\ \mathsf{Until}\ \mathsf{Blue}(x)$
- $(n_3, n_4) \vDash_{\omega_1} \mathsf{Next}(x = y)$
- $() \vDash_{w_0} \exists x.\mathsf{Next}(\mathsf{Blue}(x))$

Example – QLTL



- $n_1 \vDash_{\omega_0} \mathsf{Next}(\mathsf{Blue}(x))$
- $n_0 \vDash_{\omega_0} \neg\mathsf{Next}(\mathsf{Red}(x))$
- $n_2 \vDash_{\omega_0} \mathsf{Red}(x)\ \mathsf{Until}\ \mathsf{Blue}(x)$
- $(n_3, n_4) \vDash_{\omega_1} \mathsf{Next}(x = y)$
- $() \vDash_{w_0} \exists x.\mathsf{Next}(\mathsf{Blue}(x))$
- $(n_1, n_2) \vDash_{\omega_0} (\neg(x = y))\ \mathsf{Until}\ (x = y)$

Graph formulae

- We can define formulae that capture structural aspects of the graph:

$$\textbf{loop}(e) \quad := \quad s(e) =_{\mathsf{N}} t(e),$$

Graph formulae

- We can define formulae that capture structural aspects of the graph:

$$\begin{aligned} \textbf{loop}(e) &:= s(e) =_{\mathsf{N}} t(e), \\ \textbf{hasLoop}(n) &:= \exists_{\mathsf{E}} e.s(e) =_{\mathsf{N}} n \wedge \textbf{loop}(e), \end{aligned}$$

Graph formulae

- We can define formulae that capture structural aspects of the graph:

$$\begin{aligned} \textbf{loop}(e) &:= s(e) =_{\mathsf{N}} t(e), \\ \textbf{hasLoop}(n) &:= \exists_{\mathsf{E}} e.s(e) =_{\mathsf{N}} n \wedge \textbf{loop}(e), \\ \textbf{composable}(x, y) &:= t(x) =_{\mathsf{N}} s(y) \end{aligned}$$

Graph formulae

- We can define formulae that capture structural aspects of the graph:

$$
\begin{aligned}
\textbf{loop}(e) &:= s(e) =_{\mathsf{N}} t(e), \\
\textbf{hasLoop}(n) &:= \exists_{\mathsf{E}} e.s(e) =_{\mathsf{N}} n \wedge \textbf{loop}(e), \\
\textbf{composable}(x,y) &:= t(x) =_{\mathsf{N}} s(y) \\
\textbf{haveComposition}(x,y) &:= \textbf{composable}(x,y)
\end{aligned}
$$

Graph formulae

- We can define formulae that capture structural aspects of the graph:

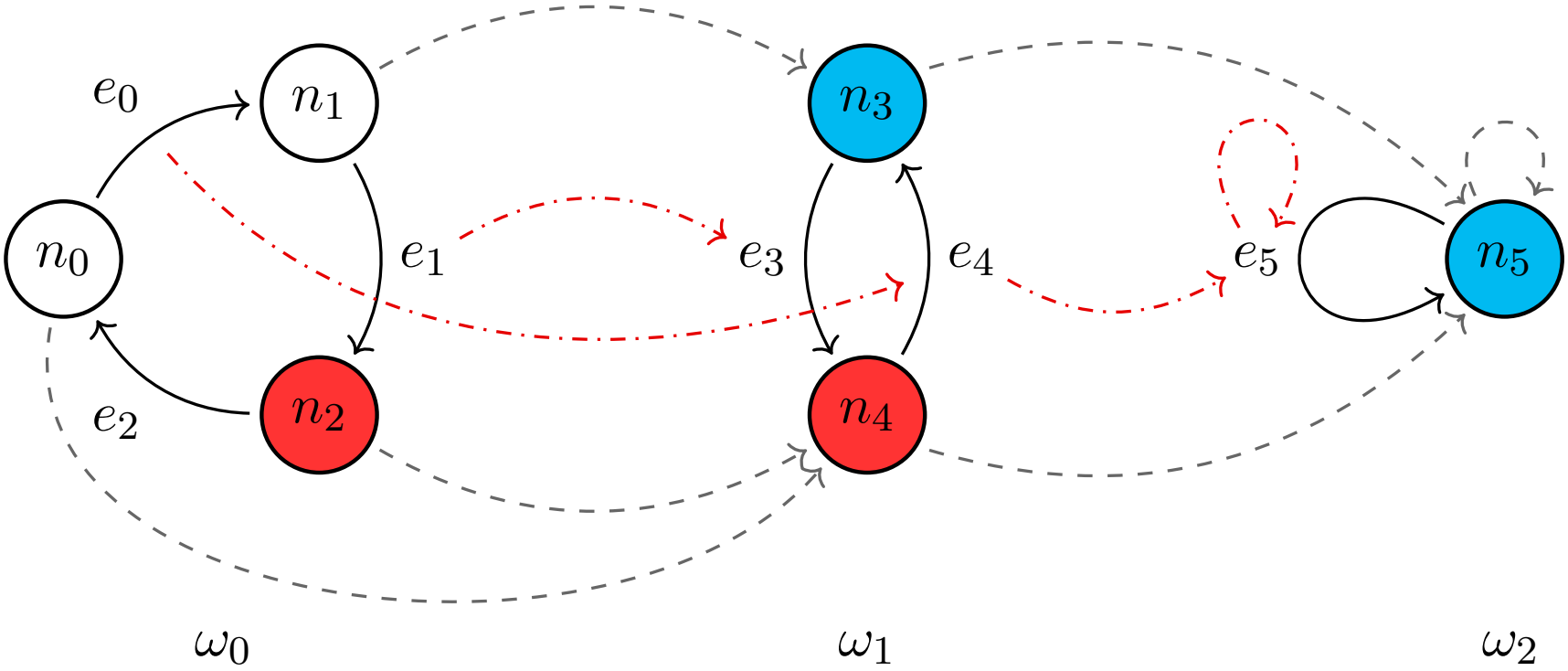
$$
\begin{aligned}
\textbf{loop}(e) &:= s(e) =_{\mathsf{N}} t(e), \\
\textbf{hasLoop}(n) &:= \exists_{\mathsf{E}} e.s(e) =_{\mathsf{N}} n \wedge \textbf{loop}(e), \\
\textbf{composable}(x,y) &:= t(x) =_{\mathsf{N}} s(y) \\
\textbf{haveComposition}(x,y) &:= \textbf{composable}(x,y) \\
&\wedge \quad \exists_{\mathsf{E}} e.(s(x) =_{\mathsf{N}} s(e) \wedge t(e) =_{\mathsf{N}} t(y))
\end{aligned}
$$

Graph formulae

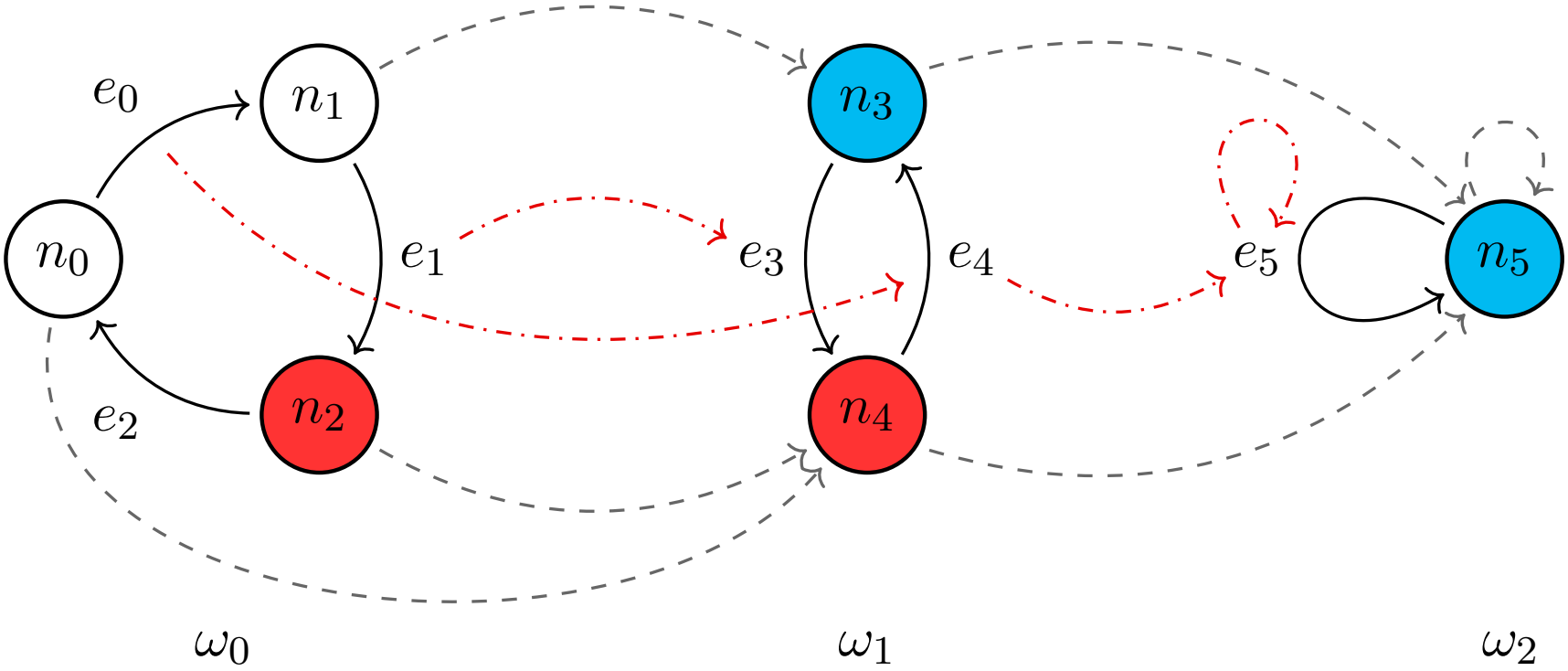
- We can define formulae that capture structural aspects of the graph:

$$
\begin{aligned}
\textbf{loop}(e) &:= s(e) =_{\mathsf{N}} t(e), \\
\textbf{hasLoop}(n) &:= \exists_{\mathsf{E}} e.s(e) =_{\mathsf{N}} n \wedge \textbf{loop}(e), \\
\textbf{composable}(x,y) &:= t(x) =_{\mathsf{N}} s(y) \\
\textbf{haveComposition}(x,y) &:= \textbf{composable}(x,y) \\
&\wedge \quad \exists_{\mathsf{E}} e.(s(x) =_{\mathsf{N}} s(e) \wedge t(e) =_{\mathsf{N}} t(y)) \\
\textbf{adjacent}(x,y) &:= \exists_{\mathsf{E}} e.((s(e) =_{\mathsf{N}} x \wedge t(e) =_{\mathsf{N}} y) \\
&\qquad \vee \ (t(e) =_{\mathsf{N}} x \wedge s(e) =_{\mathsf{N}} y))
\end{aligned}
$$

Example – QLTL on graphs

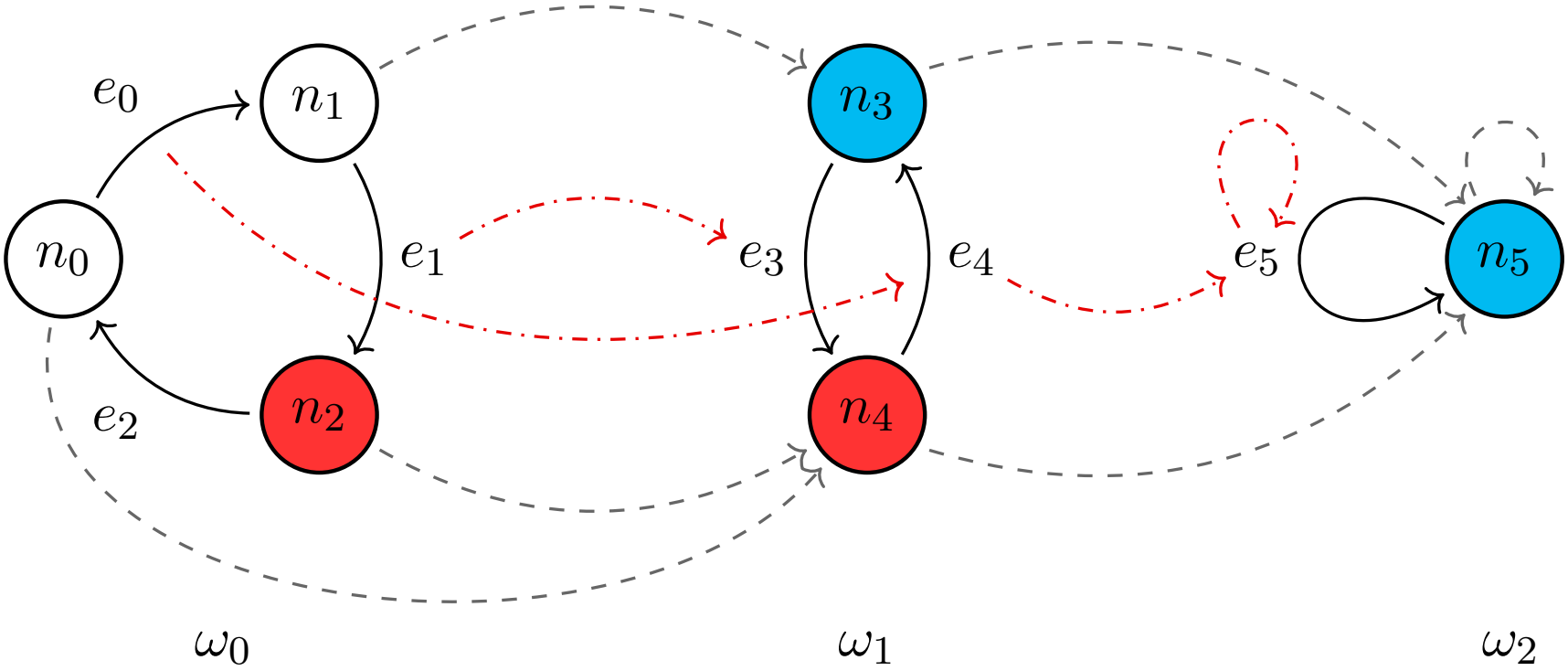


Example – QLTL on graphs



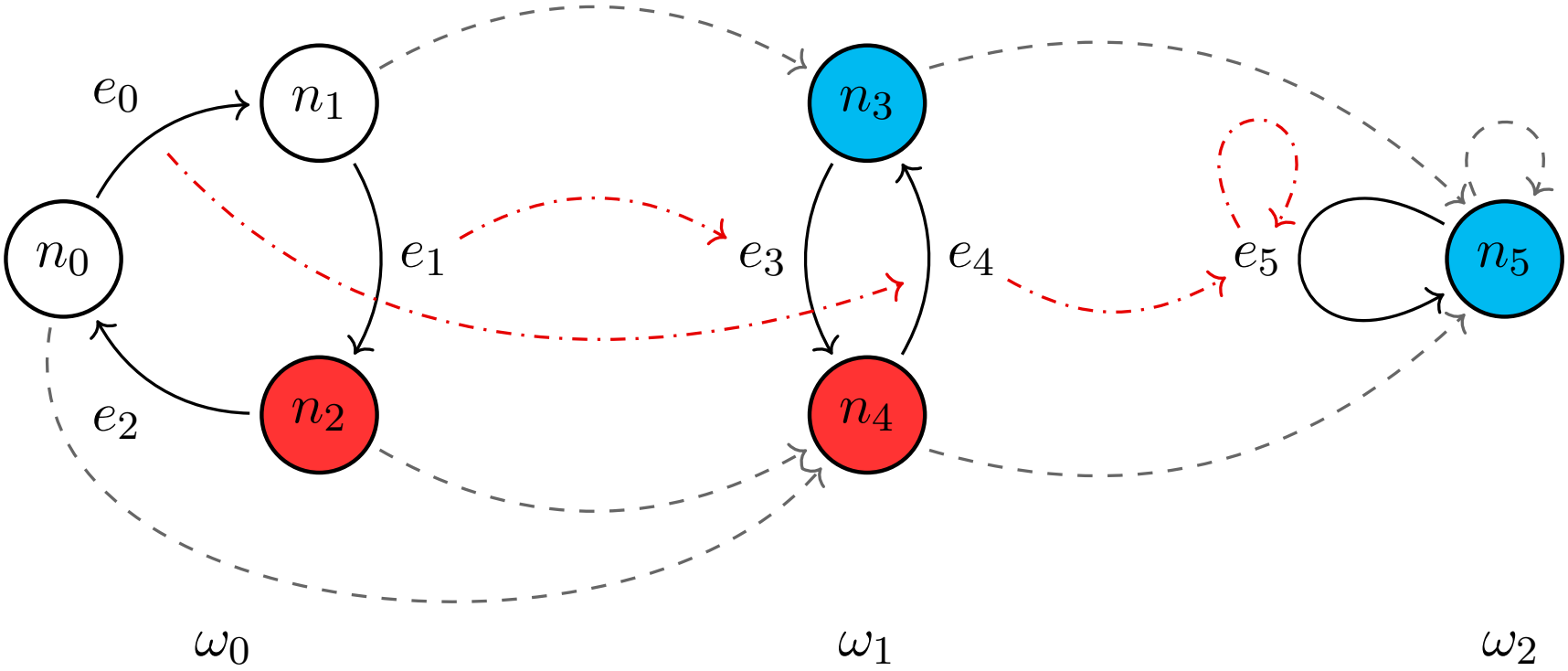
- $e_4 \vDash_{\omega_1} \mathsf{Next}(\mathbf{loop}(x))$

Example – QLTL on graphs



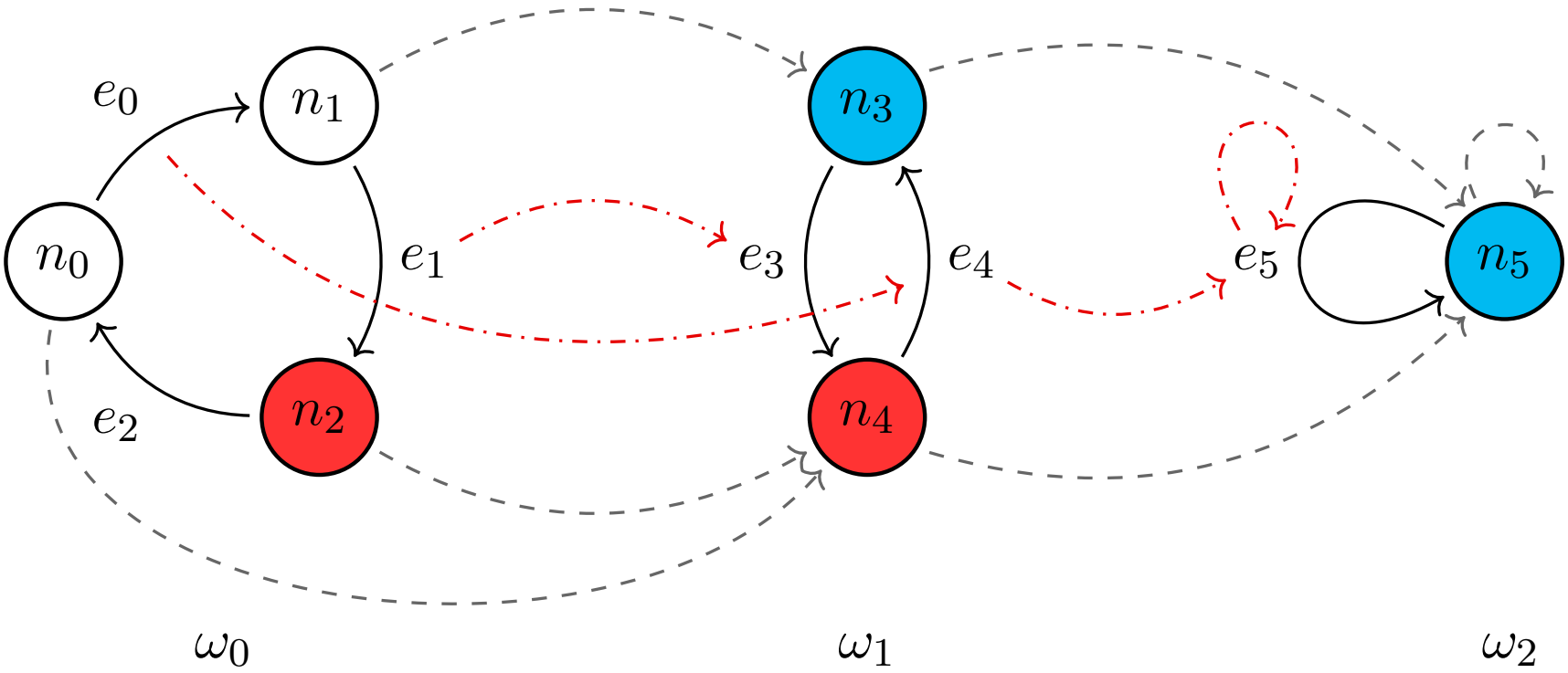
- $e_4 \models_{\omega_1} \mathsf{Next}(\mathbf{loop}(x))$
- $e_3 \models_{\omega_1} \neg\mathsf{Next}(\mathbf{loop}(x))$

Example – QLTL on graphs



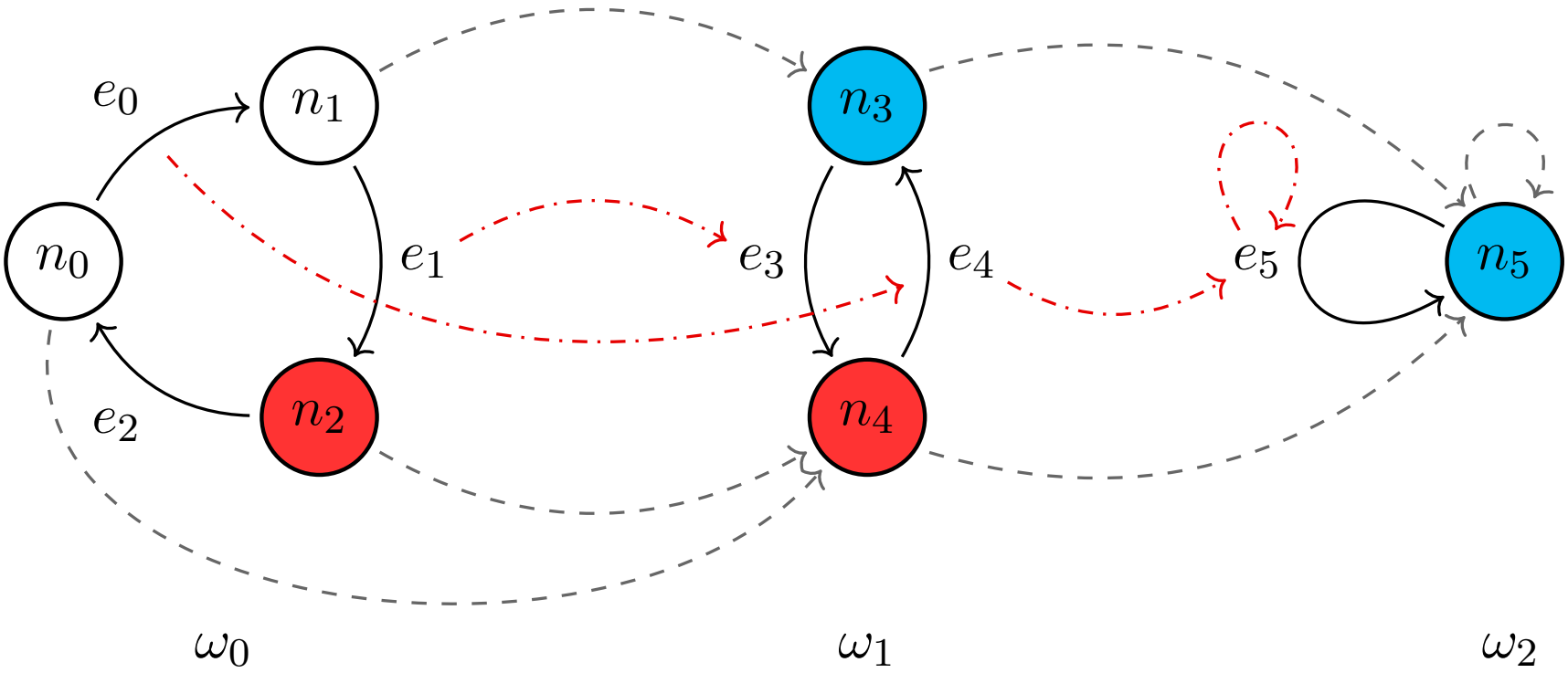
- $e_4 \models_{\omega_1} \mathsf{Next}(\mathbf{loop}(x))$
- $e_3 \models_{\omega_1} \neg\mathsf{Next}(\mathbf{loop}(x))$
- $(e_3, e_4) \models_{\omega_0} \mathbf{composable}(x, y)$

Example – QLTL on graphs



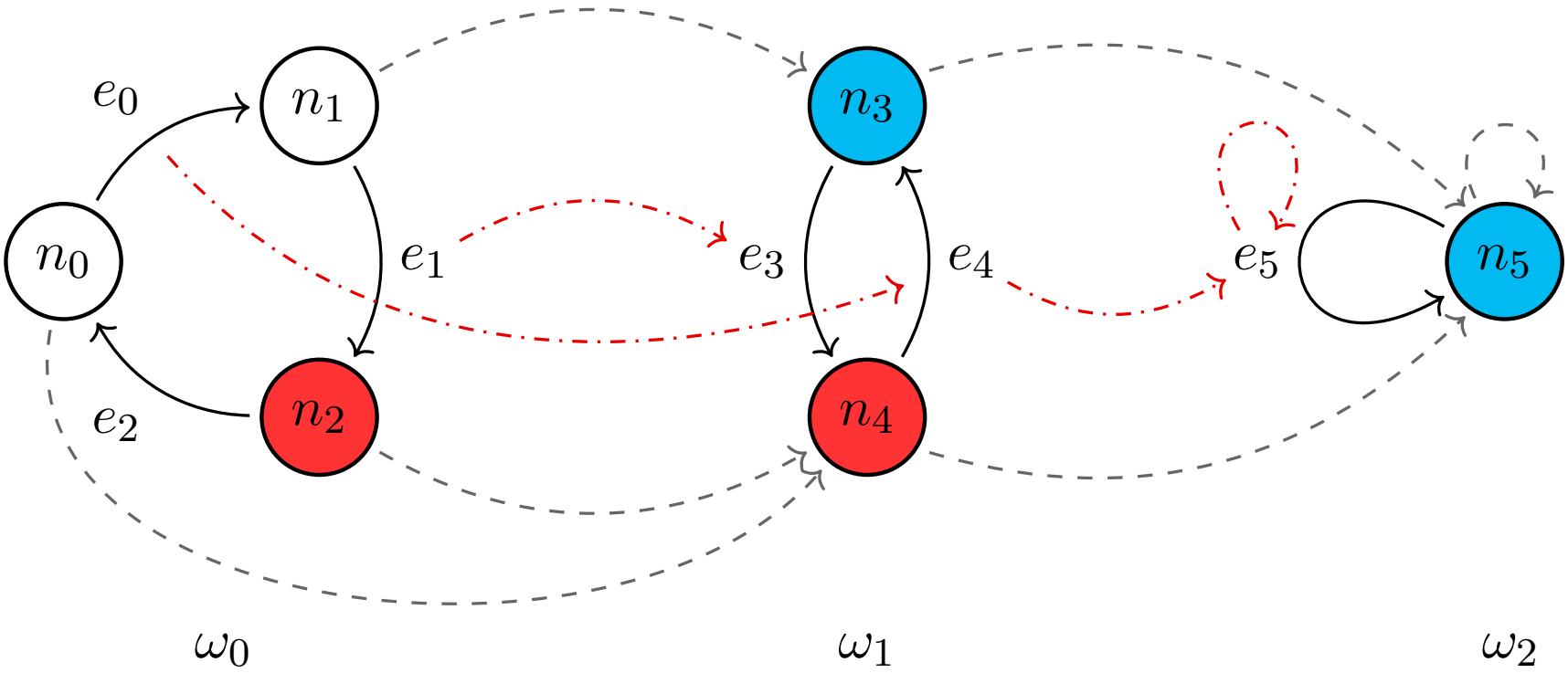
- $e_4 \vDash_{\omega_1} \mathsf{Next}(\textbf{loop}(x))$
- $e_3 \vDash_{\omega_1} \neg\mathsf{Next}(\textbf{loop}(x))$
- $(e_3, e_4) \vDash_{\omega_0} \textbf{composable}(x, y)$
- $(n_0, n_2) \vDash_{\omega_0} \textbf{adjacent}(x, y)$

Example – QLTL on graphs



- $e_4 \vDash_{\omega_1} \mathsf{Next}(\textbf{loop}(x))$
- $e_3 \vDash_{\omega_1} \neg\mathsf{Next}(\textbf{loop}(x))$
- $(e_3, e_4) \vDash_{\omega_0} \textbf{composable}(x, y)$
- $(n_0, n_2) \vDash_{\omega_0} \textbf{adjacent}(x, y)$
- $(n_0, n_2) \nvDash_{\omega_0} \mathsf{O}\textbf{adjacent}(x, y)$

Example – QLTL on graphs



- $e_4 \vDash_{\omega_1} \mathsf{Next}(\textbf{loop}(x))$
- $e_3 \vDash_{\omega_1} \neg\mathsf{Next}(\textbf{loop}(x))$
- $(e_3, e_4) \vDash_{\omega_0} \textbf{composable}(x, y)$
- $(n_0, n_2) \vDash_{\omega_0} \textbf{adjacent}(x, y)$
- $(n_0, n_2) \nvDash_{\omega_0} \mathsf{O}\textbf{adjacent}(x, y)$
- $e_0 \vDash_{\omega_0} \Diamond\textbf{loop}(x)$

Positive normal forms for QLTL

- PNF: a standard presentation for temporal logics

Positive normal forms for QLTL

- PNF: a standard presentation for temporal logics
- Usually given to simplify model checking and for fixpoint semantics

Positive normal forms for QLTL

- PNF: a standard presentation for temporal logics
- Usually given to simplify model checking and for fixpoint semantics
- PNFs are essential to work in a constructive proof assistant

Positive normal forms for QLTL

- PNF: a standard presentation for temporal logics
- Usually given to simplify model checking and for fixpoint semantics
- PNFs are essential to work in a constructive proof assistant
- PNF for QLTL:

$$\phi := \psi \mid \neg\psi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \exists_{\mathsf{E}} x.\phi \mid \exists_{\mathsf{N}} x.\phi \mid \forall_{\mathsf{E}} x.\phi \mid \forall_{\mathsf{N}} x.\phi$$

Positive normal forms for QLTL

- PNF: a standard presentation for temporal logics
- Usually given to simplify model checking and for fixpoint semantics
- PNFs are essential to work in a constructive proof assistant
- PNF for QLTL:

$$\phi := \psi \mid \neg\psi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \exists_{\mathsf{E}} x.\phi \mid \exists_{\mathsf{N}} x.\phi \mid \forall_{\mathsf{E}} x.\phi \mid \forall_{\mathsf{N}} x.\phi$$
$$\mid \mathsf{Next}(\phi) \mid \phi_1 \mathsf{Until} \phi_2 \mid \phi_1 \mathsf{WUntil} \phi_2$$

Positive normal forms for QLTL

- PNF: a standard presentation for temporal logics
- Usually given to simplify model checking and for fixpoint semantics
- PNFs are essential to work in a constructive proof assistant
- PNF for QLTL:

$$\phi := \psi \mid \neg\psi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \exists_{\mathsf{E}} x.\phi \mid \exists_{\mathsf{N}} x.\phi \mid \forall_{\mathsf{E}} x.\phi \mid \forall_{\mathsf{N}} x.\phi$$

$$\mid \mathsf{Next}(\phi) \mid \phi_1 \mathsf{Until} \phi_2 \mid \phi_1 \mathsf{WUntil} \phi_2 \mid \underline{\mathsf{NextF}}(\phi) \mid \phi_1 \underline{\mathsf{UntilF}} \phi_2 \mid \phi_1 \underline{\mathsf{WUntilF}} \phi_2$$

Positive normal forms for QLTL

- PNF: a standard presentation for temporal logics
- Usually given to simplify model checking and for fixpoint semantics
- PNFs are essential to work in a constructive proof assistant
- PNF for QLTL:

$$\phi := \psi \mid \neg\psi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \exists_{\mathsf{E}} x.\phi \mid \exists_{\mathsf{N}} x.\phi \mid \forall_{\mathsf{E}} x.\phi \mid \forall_{\mathsf{N}} x.\phi$$

$$\mid \mathsf{Next}(\phi) \mid \phi_1 \mathsf{Until} \phi_2 \mid \phi_1 \mathsf{WUntil} \phi_2 \mid \underline{\mathsf{NextF}}(\phi) \mid \phi_1 \underline{\mathsf{UntilF}} \phi_2 \mid \phi_1 \underline{\mathsf{WUntilF}} \phi_2$$

- Intuition: *universal counterparts* to the previous operators

$$\begin{aligned} \neg\mathsf{Next}(\phi) &\equiv \mathsf{NextF}(\neg\phi) \\ \neg(\phi_1 \mathsf{Until} \phi_2) &\equiv (\neg\phi_2)\mathsf{WUntilF}(\neg\phi_1 \wedge \neg\phi_2) \\ \neg(\phi_1 \mathsf{WUntil} \phi_2) &\equiv (\neg\phi_2)\mathsf{UntilF}(\neg\phi_1 \wedge \neg\phi_2) \end{aligned}$$

Positive normal forms for QLTL

- PNF: a standard presentation for temporal logics
- Usually given to simplify model checking and for fixpoint semantics
- PNFs are essential to work in a constructive proof assistant
- PNF for QLTL:

$$\phi := \psi \mid \neg\psi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \exists_{\mathsf{E}} x.\phi \mid \exists_{\mathsf{N}} x.\phi \mid \forall_{\mathsf{E}} x.\phi \mid \forall_{\mathsf{N}} x.\phi$$

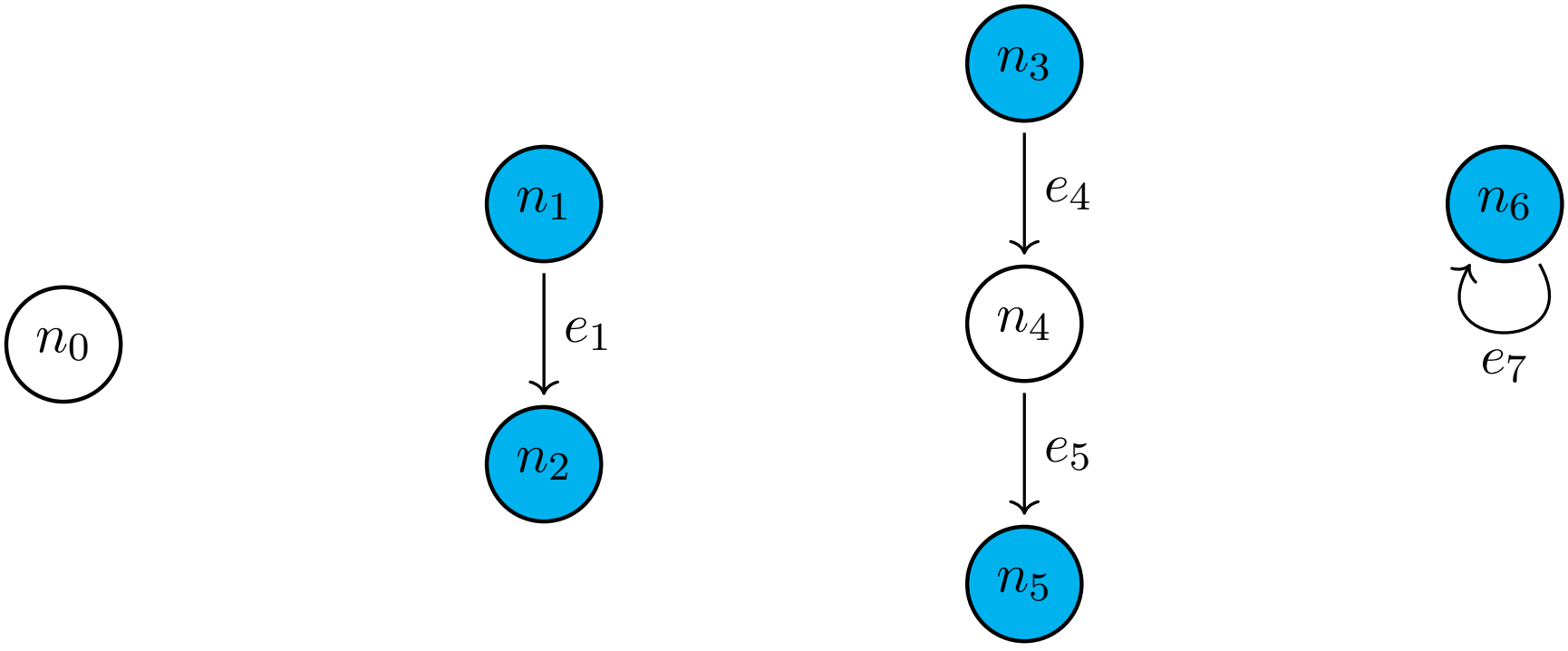
$$\mid \mathsf{Next}(\phi) \mid \phi_1 \mathsf{Until} \phi_2 \mid \phi_1 \mathsf{WUntil} \phi_2 \mid \underline{\mathsf{NextF}}(\phi) \mid \phi_1 \underline{\mathsf{UntilF}} \phi_2 \mid \phi_1 \underline{\mathsf{WUntilF}} \phi_2$$

- Intuition: *universal counterparts* to the previous operators

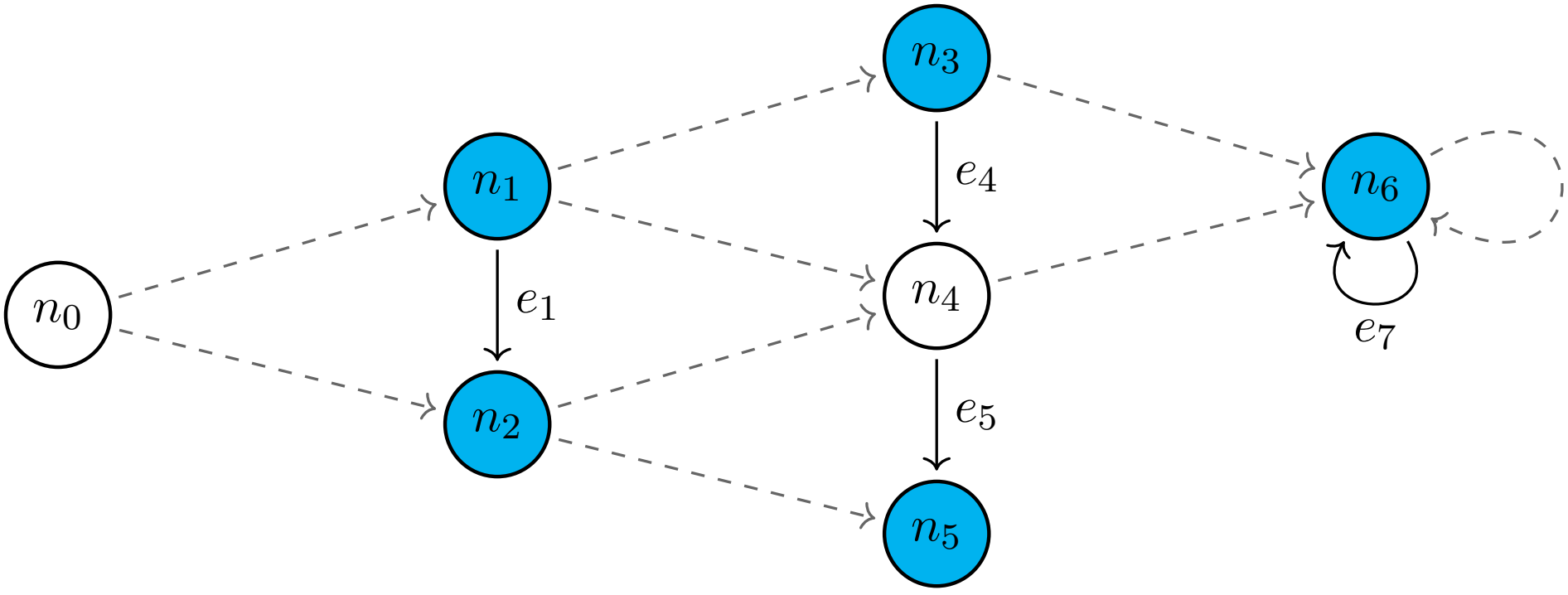
$$\begin{aligned} \neg\mathsf{Next}(\phi) &\equiv \mathsf{NextF}(\neg\phi) \\ \neg(\phi_1 \mathsf{Until} \phi_2) &\equiv (\neg\phi_2)\mathsf{WUntilF}(\neg\phi_1 \wedge \neg\phi_2) \\ \neg(\phi_1 \mathsf{WUntil} \phi_2) &\equiv (\neg\phi_2)\mathsf{UntilF}(\neg\phi_1 \wedge \neg\phi_2) \end{aligned}$$

- Become particularly useful to treat duplicating relations

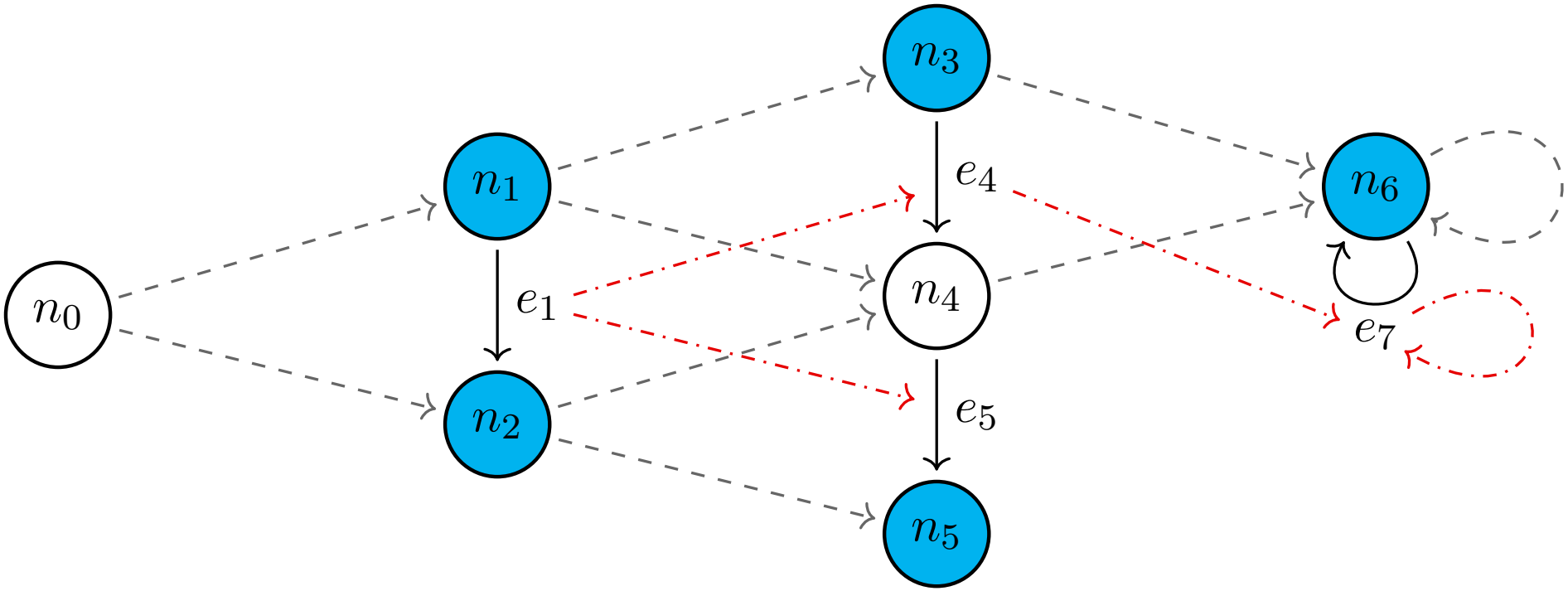
Example – Duplicating relations



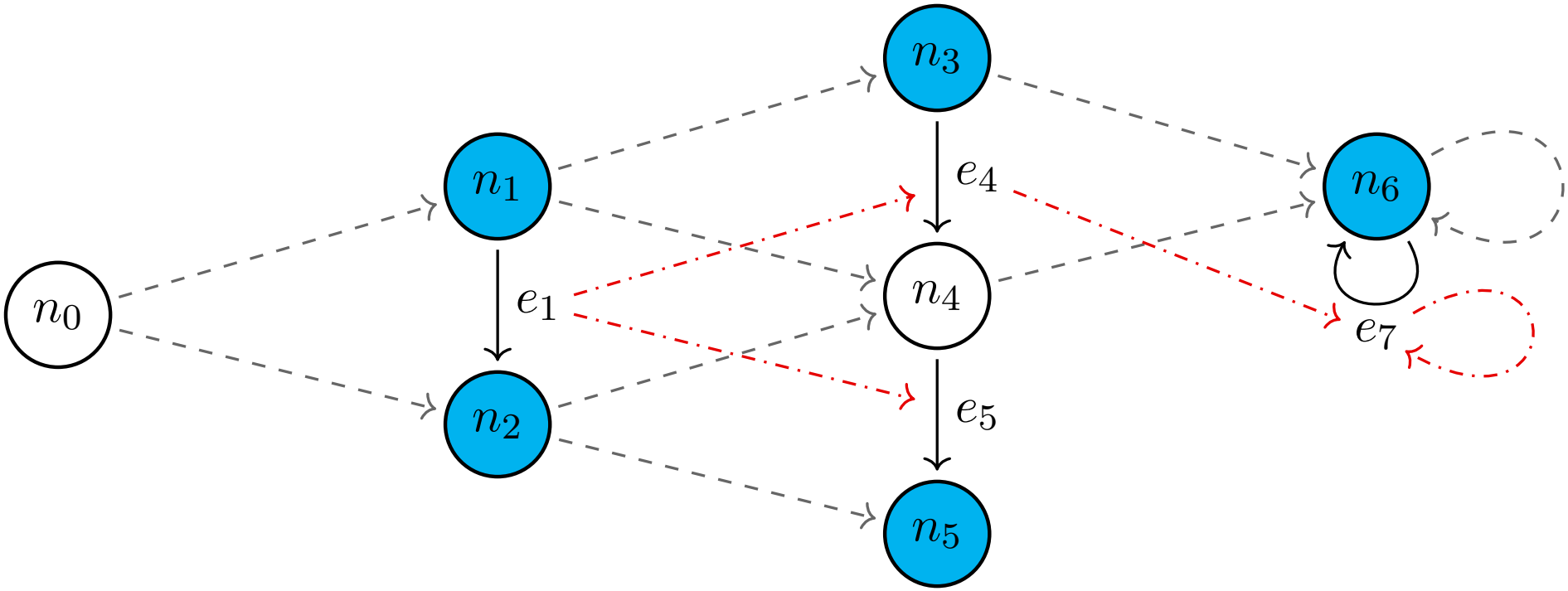
Example – Duplicating relations



Example – Duplicating relations

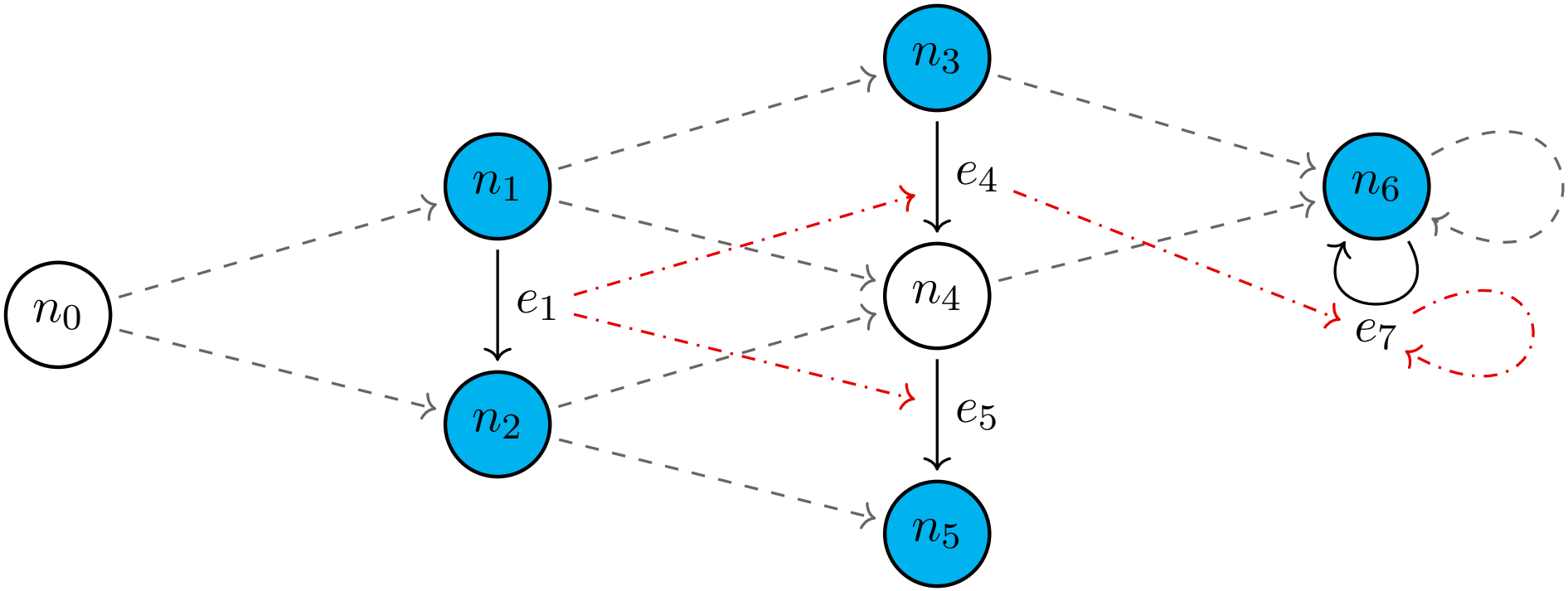


Example – Duplicating relations



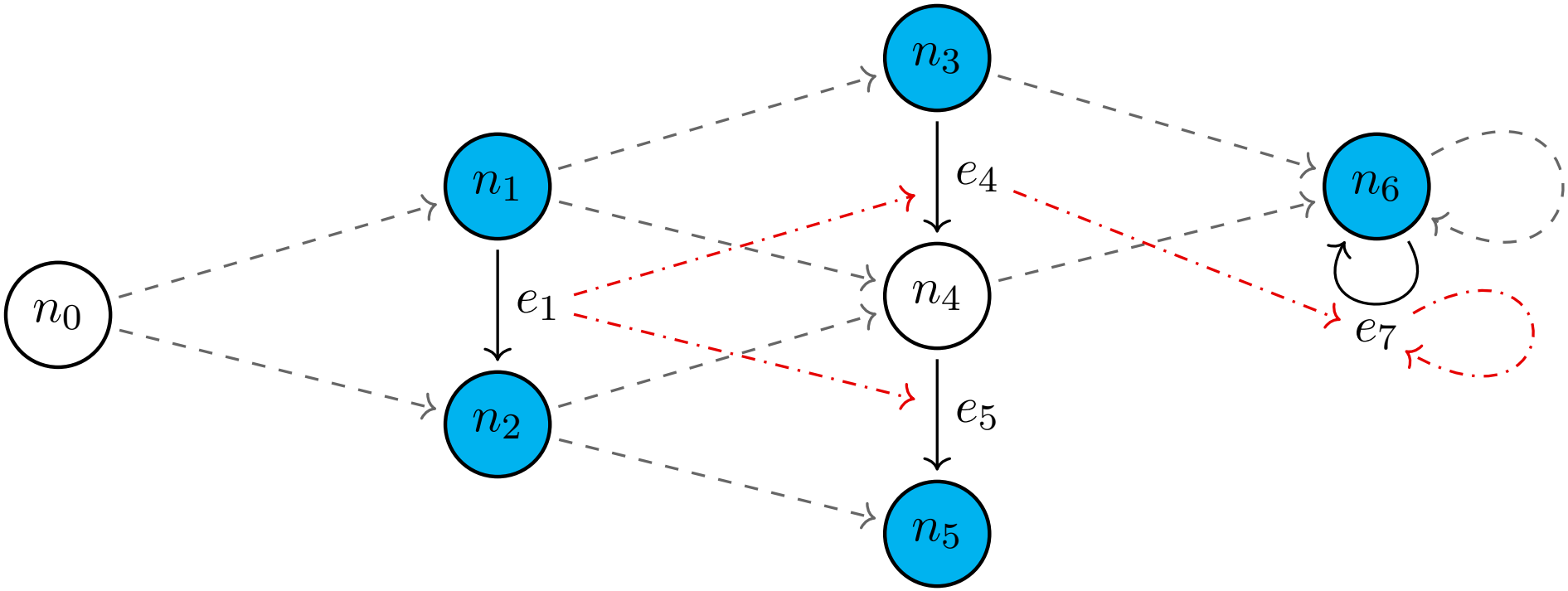
- $n_0 \vDash_{\omega_0} \mathsf{NextF}(\mathsf{Blue}(x))$

Example – Duplicating relations



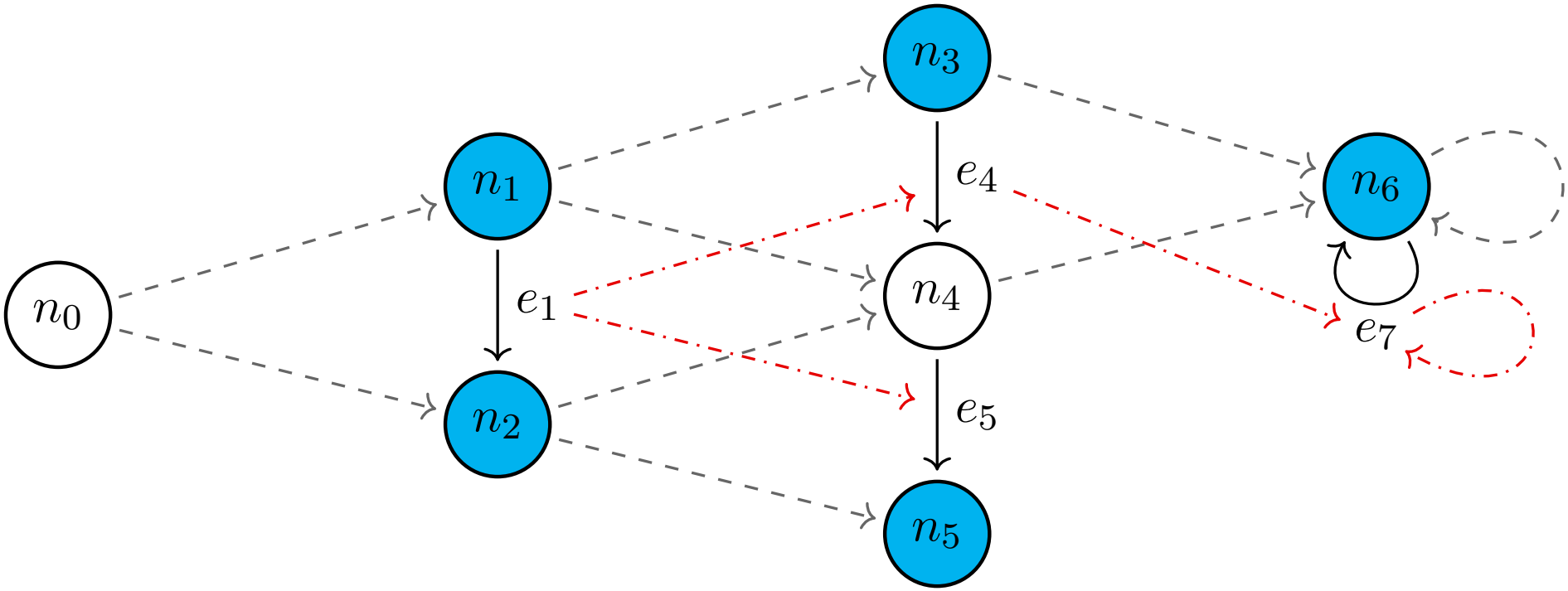
- $n_0 \models_{\omega_0} \mathsf{NextF}(\mathsf{Blue}(x))$
- $n_1 \not\models_{\omega_1} \mathsf{NextF}(\mathsf{Blue}(x))$

Example – Duplicating relations



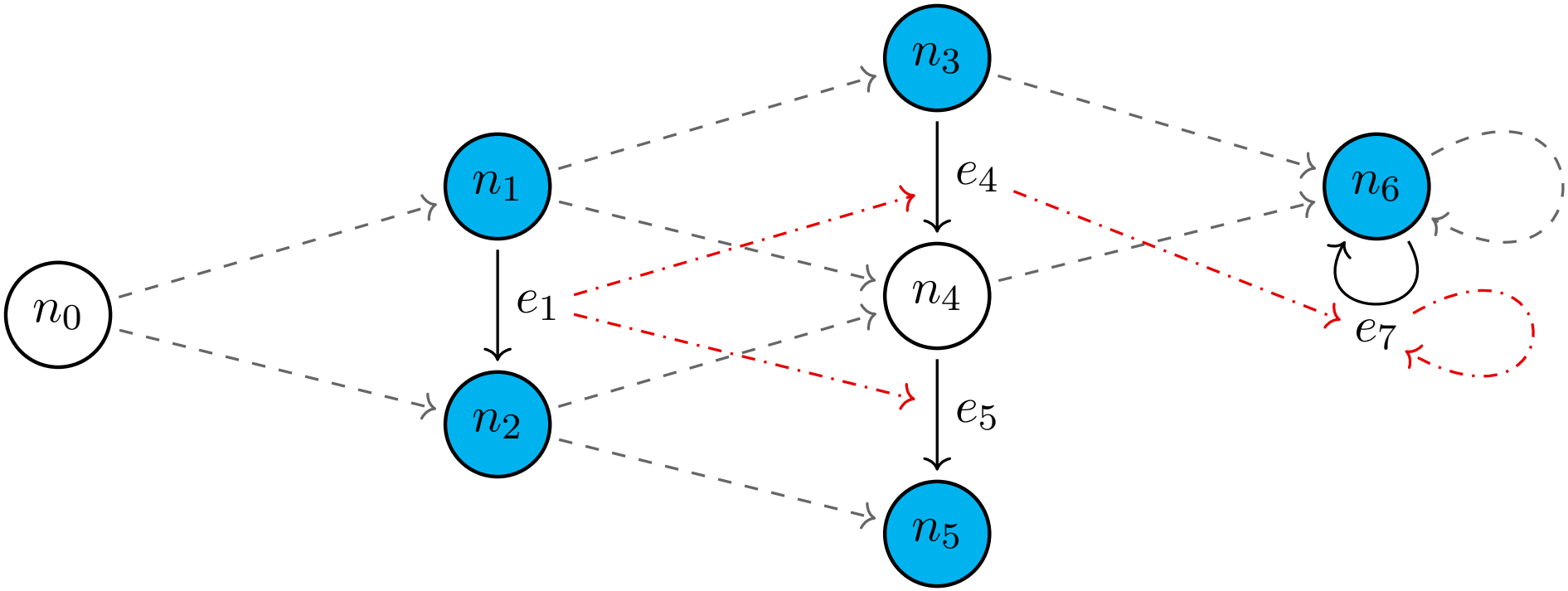
- $n_0 \models_{\omega_0} \mathsf{NextF}(\mathsf{Blue}(x))$
- $n_1 \not\models_{\omega_1} \mathsf{NextF}(\mathsf{Blue}(x))$
- $e_5 \models_{\omega_1} \mathsf{NextF}(\mathbf{loop}(x))$

Example – Duplicating relations



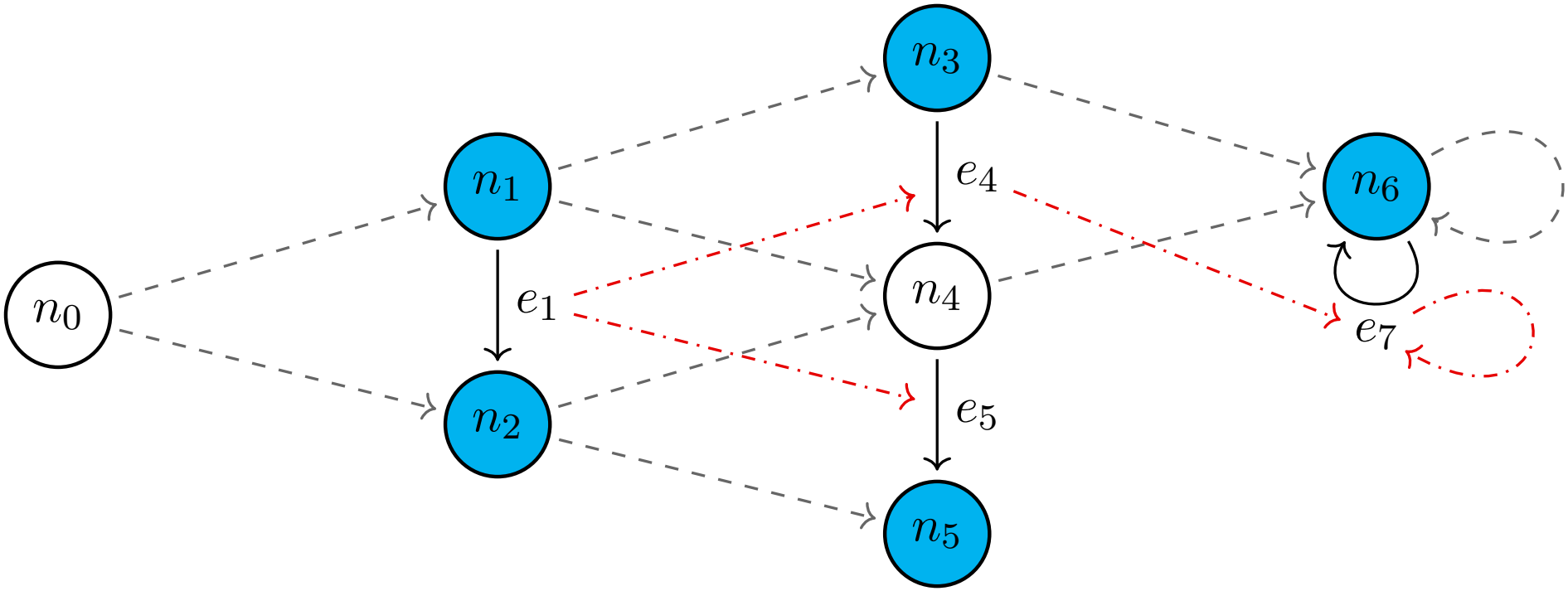
- $n_0 \vDash_{\omega_0} \mathsf{NextF}(\mathsf{Blue}(x))$
- $n_1 \nvDash_{\omega_1} \mathsf{NextF}(\mathsf{Blue}(x))$
- $e_5 \vDash_{\omega_1} \mathsf{NextF}(\mathbf{loop}(x))$
- $e_1 \vDash_{\omega_1} \mathsf{Blue}(s(x))\ \mathsf{Until}\ (\mathbf{loop}(x))$

Example – Duplicating relations



- $n_0 \vDash_{\omega_0}$ NextF(Blue(x))
- $n_1 \nvDash_{\omega_1}$ NextF(Blue(x))
- $e_5 \vDash_{\omega_1}$ NextF(**loop**(x))
- $e_1 \vDash_{\omega_1}$ Blue($s(x)$) Until (**loop**(x))
- $e_1 \nvDash_{\omega_1}$ Blue($s(x)$) UntilF (**loop**(x))

Example – Duplicating relations



- $n_0 \vDash_{\omega_0} \mathsf{NextF}(\mathsf{Blue}(x))$
- $n_1 \nvDash_{\omega_1} \mathsf{NextF}(\mathsf{Blue}(x))$
- $e_5 \vDash_{\omega_1} \mathsf{NextF}(\mathbf{loop}(x))$
- $e_1 \vDash_{\omega_1} \mathsf{Blue}(s(x))\ \mathsf{Until}\ (\mathbf{loop}(x))$
- $e_1 \nvDash_{\omega_1} \mathsf{Blue}(s(x))\ \mathsf{UntilF}\ (\mathbf{loop}(x))$
- $e_4 \vDash_{\omega_1} \mathsf{Blue}(s(x))\ \mathsf{WUntilF}\ (\mathbf{false})$

Agda formalization

- Agda: *dependently typed programming language and proof assistant*

Agda formalization

- Agda: *dependently typed programming language and proof assistant*
- Can be used in practice to formalize mathematical constructions

Agda formalization

- Agda: *dependently typed programming language and proof assistant*
- Can be used in practice to formalize mathematical constructions
- Mechanization work:

Agda formalization

- Agda: *dependently typed programming language and proof assistant*
- Can be used in practice to formalize mathematical constructions
- Mechanization work:

1. A formalization of categorical QLTL and its models in Agda

Agda formalization

- Agda: *dependently typed programming language and proof assistant*
- Can be used in practice to formalize mathematical constructions
- Mechanization work:
  1. A formalization of categorical QLTL and its models in Agda
  2. Categorical semantics formalized using the agda-categories library

Agda formalization

- Agda: *dependently typed programming language and proof assistant*
- Can be used in practice to formalize mathematical constructions
- Mechanization work:
  1. A formalization of categorical QLTL and its models in Agda
  2. Categorical semantics formalized using the agda-categories library
  3. Algebraic QLTL: worlds-as-algebras over any multi-sorted signature

Agda formalization

- Agda: *dependently typed programming language and proof assistant*
- Can be used in practice to formalize mathematical constructions
- Mechanization work:
  1. A formalization of categorical QLTL and its models in Agda
  2. Categorical semantics formalized using the agda-categories library
  3. Algebraic QLTL: worlds-as-algebras over any multi-sorted signature
  4. A classical set-based semantics without the use of categorical logic

Agda formalization

- Agda: *dependently typed programming language and proof assistant*
- Can be used in practice to formalize mathematical constructions
- Mechanization work:

1. A formalization of categorical QLTL and its models in Agda
2. Categorical semantics formalized using the agda-categories library
3. Algebraic QLTL: worlds-as-algebras over any multi-sorted signature
4. A classical set-based semantics without the use of categorical logic
5. Presentation of the *positive normal forms* of QLTL, also in Agda

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code
- *Why is a formal presentation of our logic useful?*

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code
- *Why is a formal presentation of our logic useful?*
- Formalizing constructions and semantics establishes their *correctness*

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code
- *Why is a formal presentation of our logic useful?*


Formalizing constructions and semantics establishes their *correctness*


Provides a formal setting to *test and experiment with* temporal logics

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code
- *Why is a formal presentation of our logic useful?*


Formalizing constructions and semantics establishes their *correctness*


Provides a formal setting to *test and experiment with* temporal logics


Establishes a foundation to build *verified model checkers* for QTL

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code
- *Why is a formal presentation of our logic useful?*

- Formalizing constructions and semantics establishes their *correctness*
- Provides a formal setting to *test and experiment with* temporal logics
- Establishes a foundation to build *verified model checkers* for QTL
- Gives a program to *convert* standard models into categorical ones:

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code
- *Why is a formal presentation of our logic useful?*

- Formalizing constructions and semantics establishes their *correctness*
- Provides a formal setting to *test and experiment with* temporal logics
- Establishes a foundation to build *verified model checkers* for QTL
- Gives a program to *convert* standard models into categorical ones:
  1. Define the semantics of the logic with *categorical* notions and models

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code
- *Why is a formal presentation of our logic useful?*

- Formalizing constructions and semantics establishes their *correctness*
- Provides a formal setting to *test and experiment with* temporal logics
- Establishes a foundation to build *verified model checkers* for QTL
- Gives a program to *convert* standard models into categorical ones:
  1. Define the semantics of the logic with *categorical* notions and models
  2. Provide a standard *non-categorical* transition system as model

Agda formalization

- Categorical semantics: **1388 lines** of Agda code
- Positive normal form: **1740 lines** of Agda code
- *Why is a formal presentation of our logic useful?*

- Formalizing constructions and semantics establishes their *correctness*
- Provides a formal setting to *test and experiment with* temporal logics
- Establishes a foundation to build *verified model checkers* for QTL
- Gives a program to *convert* standard models into categorical ones:
  1. Define the semantics of the logic with *categorical* notions and models
  2. Provide a standard *non-categorical* transition system as model
  3. Use the procedure `ClassicalToCategorical` to construct the categorical model so that the logic can be applied

Experience with agda-categories

https://github.com/agda/agda-categories

- The *de-facto* (non-univalent) standard *category theory library* in Agda

Experience with agda-categories

https://github.com/agda/agda-categories

- The *de-facto* (non-univalent) standard *category theory library* in Agda
- Extremely practical and flexible, no magic involved

Experience with agda-categories

https://github.com/agda/agda-categories

- The *de-facto* (non-univalent) standard *category theory library* in Agda
- Extremely practical and flexible, no magic involved
- Design choices do not necessarily get in the way of practical applications

Experience with agda-categories

https://github.com/agda/agda-categories

- The *de-facto* (non-univalent) standard *category theory library* in Agda
- ☺ Extremely practical and flexible, no magic involved
- ☺ Design choices do not necessarily get in the way of practical applications
- Main definitions used:
  - Categories, functors, natural transformations
  - **Rel**: category of sets and relations
  - Free categories generated from a quiver (`PathCategory`)
  - Presheaves, the category of (relational) presheaves is complete
  - Relational presheaves and morphisms between them

Experience with agda-categories

https://github.com/agda/agda-categories

- The *de-facto* (non-univalent) standard *category theory library* in Agda
- ☺ Extremely practical and flexible, no magic involved
- ☺ Design choices do not necessarily get in the way of practical applications
- Main definitions used:
  - Categories, functors, natural transformations
  - **Rel**: category of sets and relations
  - Free categories generated from a quiver (`PathCategory`)
  - Presheaves, the category of (relational) presheaves is complete
  - Relational presheaves and morphisms between them
- 😐 Functoriality and setoid-equality preservation can be annoying to prove

Experience with agda-categories

https://github.com/agda/agda-categories

- The *de-facto* (non-univalent) standard *category theory library* in Agda
- ☺ Extremely practical and flexible, no magic involved
- ☺ Design choices do not necessarily get in the way of practical applications
- Main definitions used:
  - Categories, functors, natural transformations
  - **Rel**: category of sets and relations
  - Free categories generated from a quiver (`PathCategory`)
  - Presheaves, the category of (relational) presheaves is complete
  - Relational presheaves and morphisms between them
- 😐 Functoriality and setoid-equality preservation can be annoying to prove
- 😐 Relatively limited use of the theorems/properties given by the library

PNF in Agda

- Formalized in Agda: PNF equivalence (using classical reasoning)

PNF in Agda

- Formalized in Agda: PNF equivalence (using classical reasoning)
- Two cases, using non-categorical semantics:

PNF in Agda

- Formalized in Agda: PNF equivalence (using classical reasoning)
- Two cases, using non-categorical semantics:
  - PNF with *partial functions* as counterpart relations

PNF in Agda

- Formalized in Agda: PNF equivalence (using classical reasoning)
- Two cases, using non-categorical semantics:
  - PNF with *partial functions* as counterpart relations
  - PNF with general relations (i.e. allow duplication of entities)

PNF in Agda

- Formalized in Agda: PNF equivalence (using classical reasoning)
- Two cases, using non-categorical semantics:
  - PNF with *partial functions* as counterpart relations
  - PNF with general relations (i.e. allow duplication of entities)
- Expansion laws and equivalences in QLTL in both settings

PNF in Agda

- Formalized in Agda: PNF equivalence (using classical reasoning)
- Two cases, using non-categorical semantics:
  - PNF with *partial functions* as counterpart relations
  - PNF with general relations (i.e. allow duplication of entities)
- Expansion laws and equivalences in QLTL in both settings

$\Rightarrow$ LTL-like expansion laws break down in the case of relations!

PNF in Agda

- Formalized in Agda: PNF equivalence (using classical reasoning)
- Two cases, using non-categorical semantics:
  - PNF with *partial functions* as counterpart relations
  - PNF with general relations (i.e. allow duplication of entities)
- Expansion laws and equivalences in QLTL in both settings

$\Rightarrow$ LTL-like expansion laws break down in the case of relations!

$\Rightarrow$ (But they can be mostly recovered in the case of partial functions.)

Conclusion and Future Work

In this work we present a counterpart-based temporal logic that can reason on the temporal evolution of algebraic structures and formalize its semantics in Agda along with results on its PNF.

- Many possible extensions of this work:

Conclusion and Future Work

In this work we present a counterpart-based temporal logic that can reason on the temporal evolution of algebraic structures and formalize its semantics in Agda along with results on its PNF.

- Many possible extensions of this work:
  - formalization of second-order QLTL to express set quantification

Conclusion and Future Work

In this work we present a counterpart-based temporal logic that can reason on the temporal evolution of algebraic structures and formalize its semantics in Agda along with results on its PNF.

- Many possible extensions of this work:
  - formalization of second-order QLTL to express set quantification
  - extending counterpart semantics to CTL, CTL* and their models

Conclusion and Future Work

In this work we present a counterpart-based temporal logic that can reason on the temporal evolution of algebraic structures and formalize its semantics in Agda along with results on its PNF.

- Many possible extensions of this work:
  - formalization of second-order QLTL to express set quantification
  - extending counterpart semantics to CTL, CTL* and their models
  - interfacing Agda with SMT solvers and model checkers for QLTL

Conclusion and Future Work

In this work we present a counterpart-based temporal logic that can reason on the temporal evolution of algebraic structures and formalize its semantics in Agda along with results on its PNF.

- Many possible extensions of this work:
  - formalization of second-order QLTL to express set quantification
  - extending counterpart semantics to CTL, CTL* and their models
  - interfacing Agda with SMT solvers and model checkers for QLTL
  - formalize syntax and models of the logic with indexed categories and morphisms between them, as in categorical logic [Jacobs, 2001]

Conclusion and Future Work

In this work we present a counterpart-based temporal logic that can reason on the temporal evolution of algebraic structures and formalize its semantics in Agda along with results on its PNF.

- Many possible extensions of this work:
  - formalization of second-order QLTL to express set quantification
  - extending counterpart semantics to CTL, CTL* and their models
  - interfacing Agda with SMT solvers and model checkers for QLTL
  - formalize syntax and models of the logic with indexed categories and morphisms between them, as in categorical logic [Jacobs, 2001]
- A study of formally-presented temporal logics is absent in the literature

Conclusion and Future Work

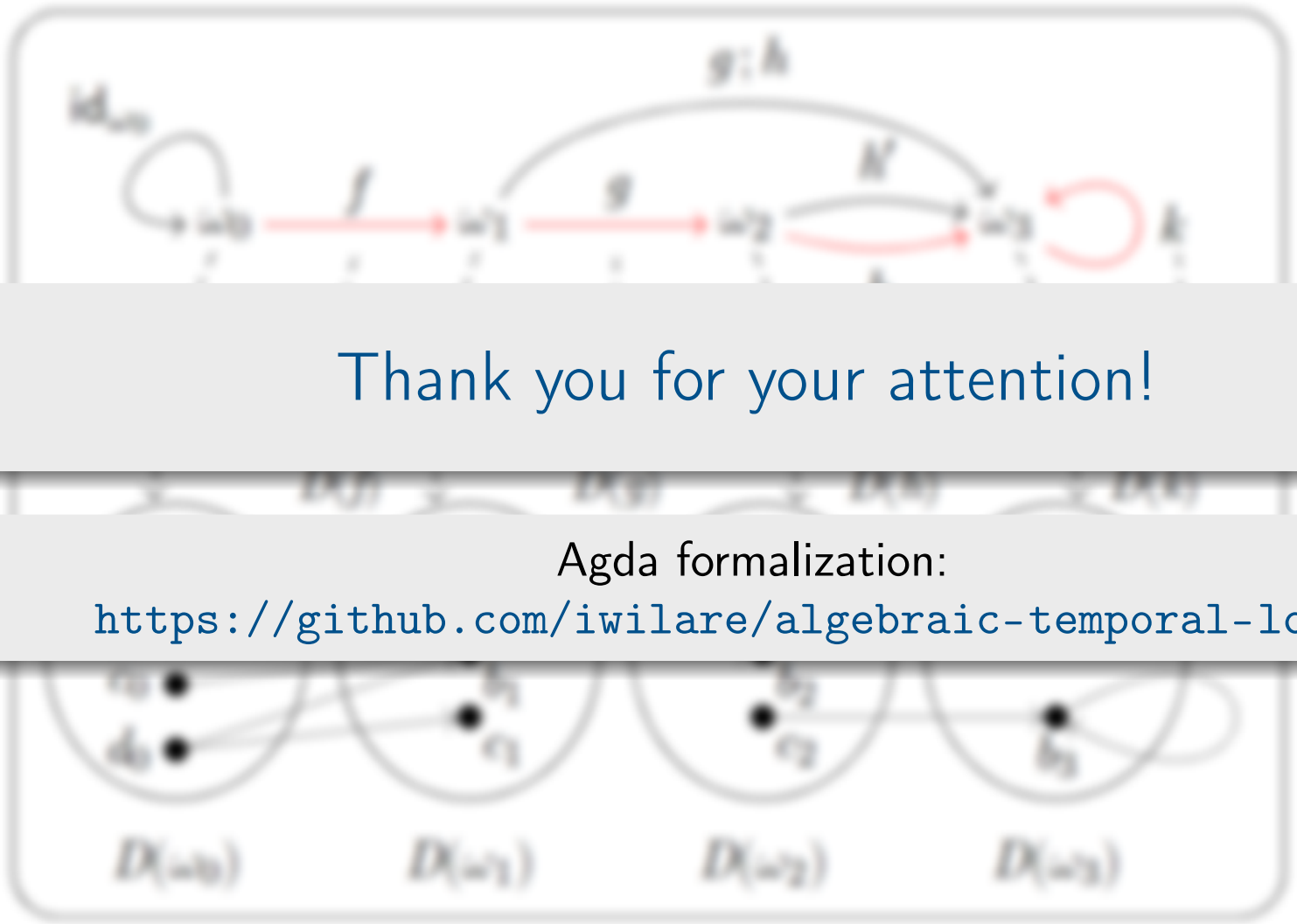
In this work we present a counterpart-based temporal logic that can reason on the temporal evolution of algebraic structures and formalize its semantics in Agda along with results on its PNF.

- Many possible extensions of this work:
  - formalization of second-order QLTL to express set quantification
  - extending counterpart semantics to CTL, CTL* and their models
  - interfacing Agda with SMT solvers and model checkers for QLTL
  - formalize syntax and models of the logic with indexed categories and morphisms between them, as in categorical logic [Jacobs, 2001]
- A study of formally-presented temporal logics is absent in the literature
- Other verified model checkers: LTL in Isabelle [Nipkow, 2013]

Conclusion and Future Work

In this work we present a counterpart-based temporal logic that can reason on the temporal evolution of algebraic structures and formalize its semantics in Agda along with results on its PNF.

- Many possible extensions of this work:
  - formalization of second-order QLTL to express set quantification
  - extending counterpart semantics to CTL, CTL* and their models
  - interfacing Agda with SMT solvers and model checkers for QLTL
  - formalize syntax and models of the logic with indexed categories and morphisms between them, as in categorical logic [Jacobs, 2001]
- A study of formally-presented temporal logics is absent in the literature
- Other verified model checkers: LTL in Isabelle [Nipkow, 2013]
- Proof searching using reflection in Agda for CTL [O'Connor, 2016]



Thank you for your attention!

Agda formalization:
https://github.com/iwilare/algebraic-temporal-logics