Specification and Verification of a Linear-Time Temporal Logic for Graph Transformation

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Contribution and Outline

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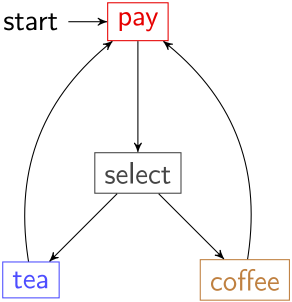

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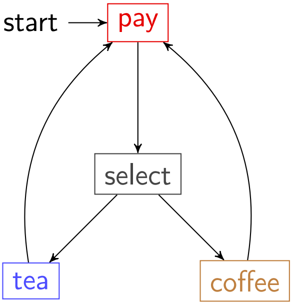

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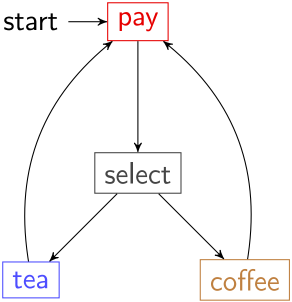

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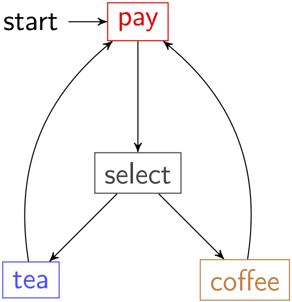

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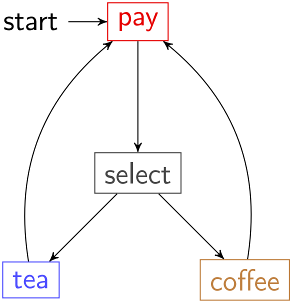

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3. Use a program to *check* that the *model* **satisfies** the formula

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Can we enrich our models to express multi-component behaviour?
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- *Yes!* Using **counterpart models** and quantified temporal logics

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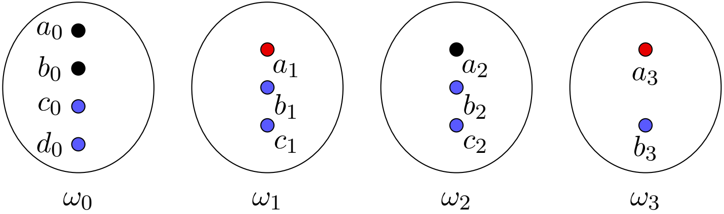

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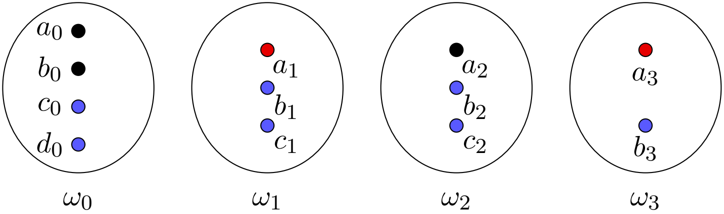

How do we represent transitions?

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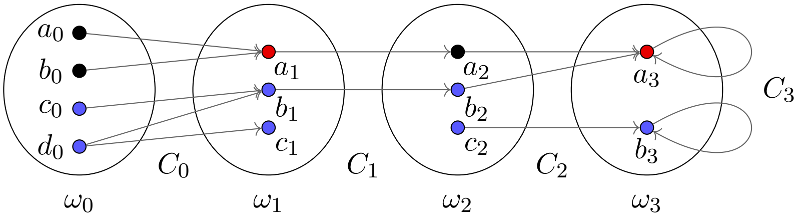

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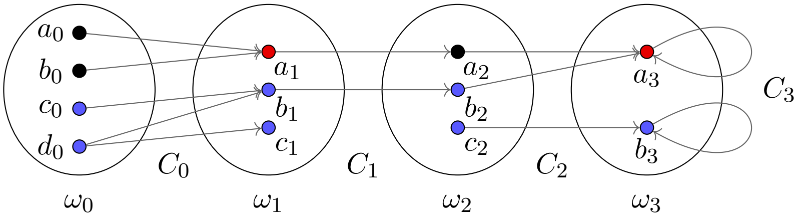

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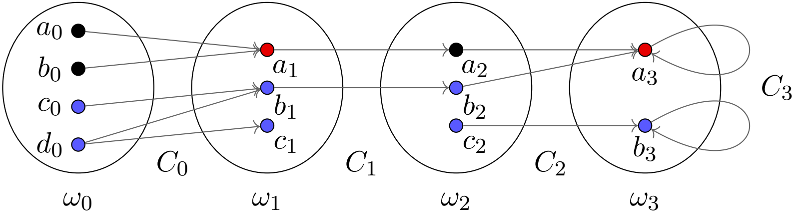

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- We call these sequences of worlds and relations **counterpart traces**

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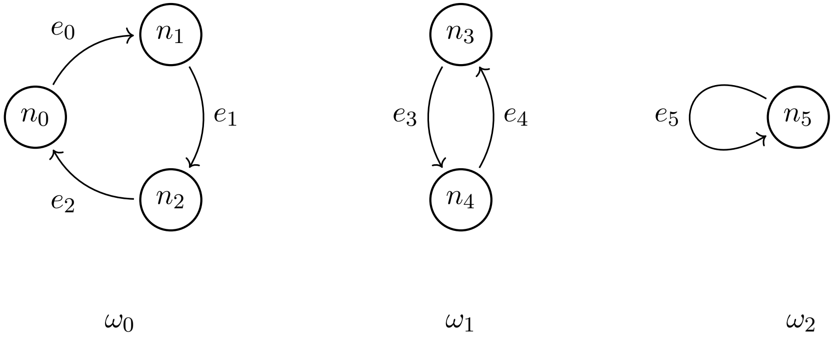

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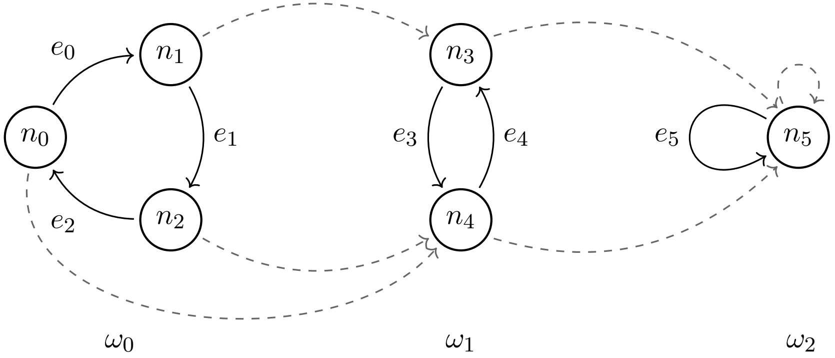

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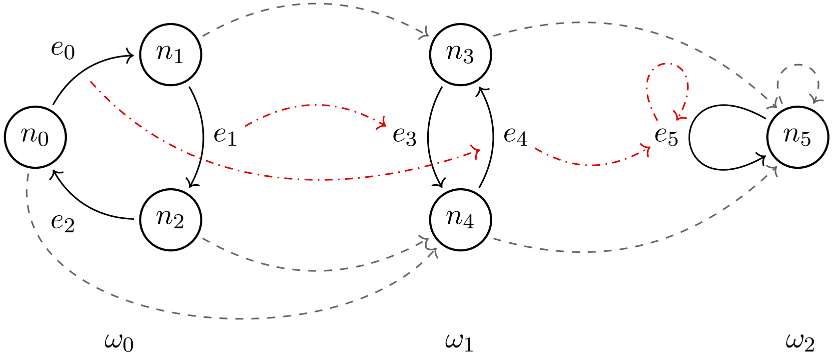

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Example – Counterpart model

$$
\begin{tikzcd}
\omega_0 \arrow[loop left, "\mathsf{id}_{\omega_0}"] \arrow[r, "f"] & \omega_1 \arrow[r, "g"] \arrow[rr, bend left, "g\,;h"] & \omega_2 \arrow[r, bend left, "h'"] \arrow[r, bend right, "h"'] & \omega_3 \arrow[loop right, "k"]
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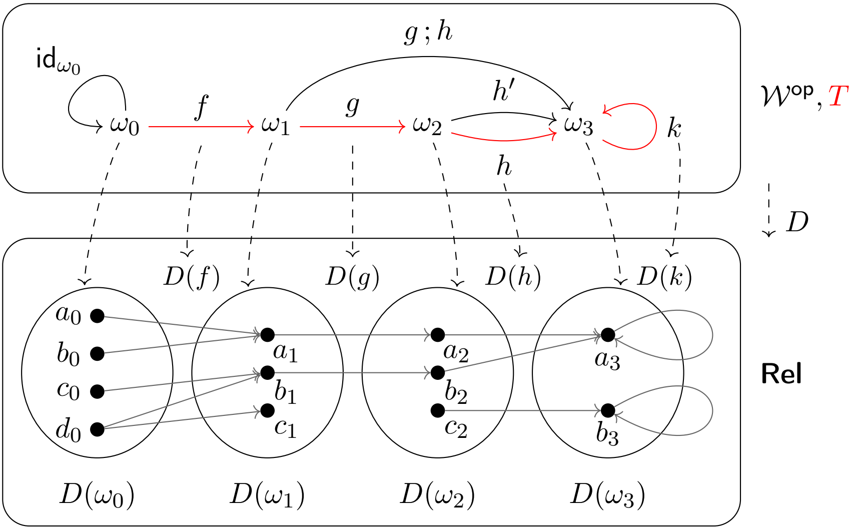

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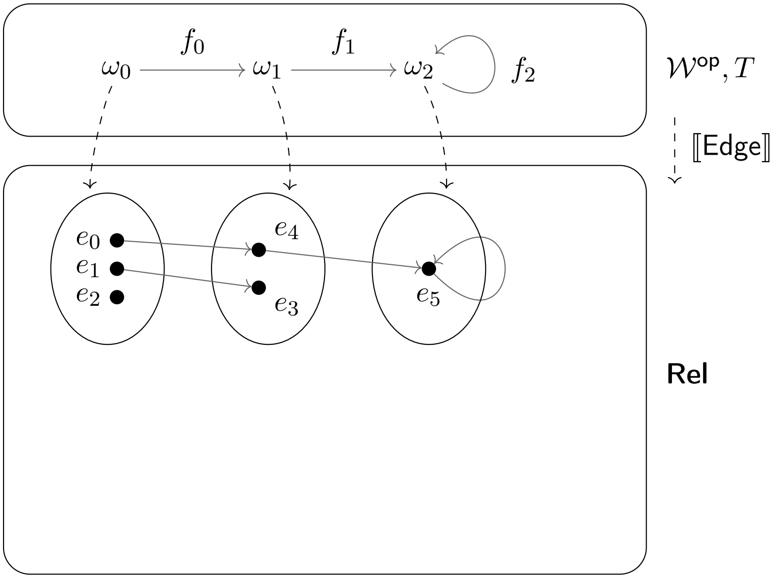

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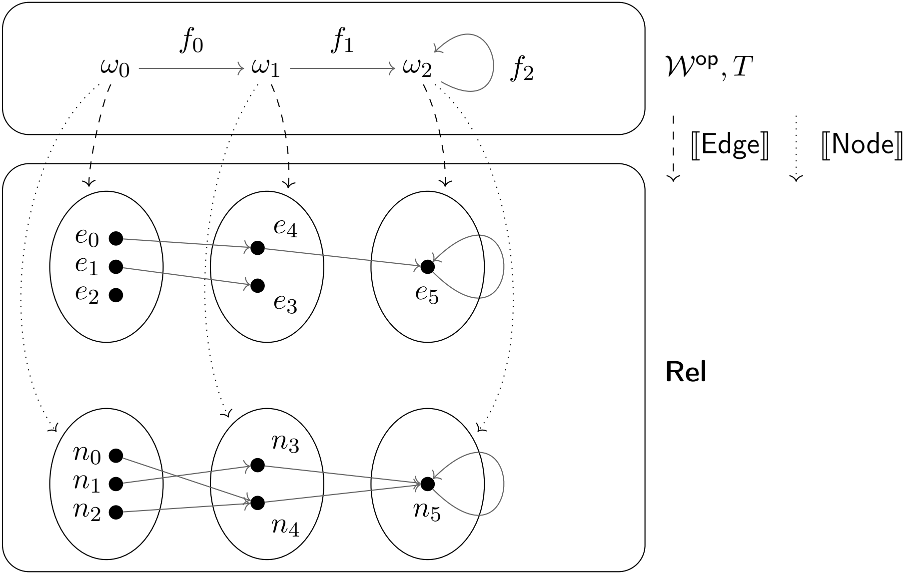

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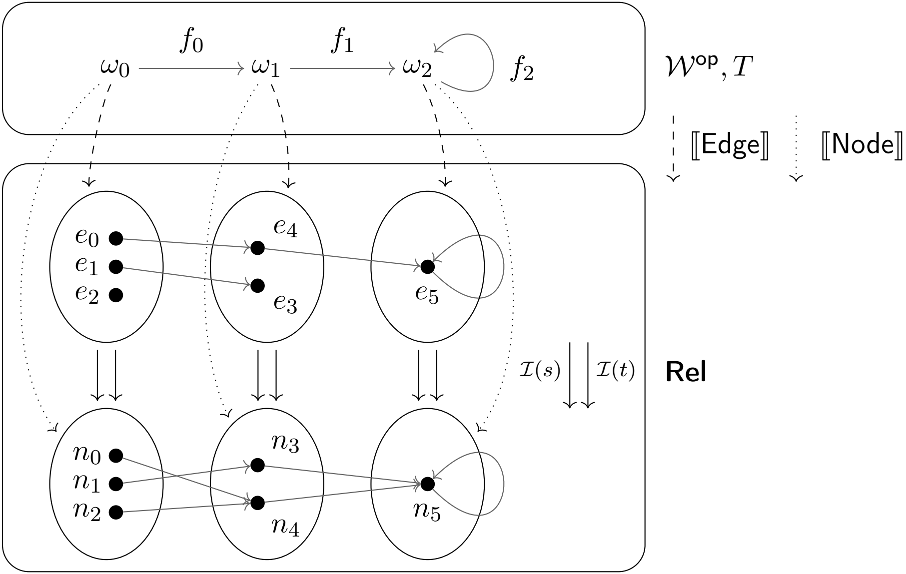

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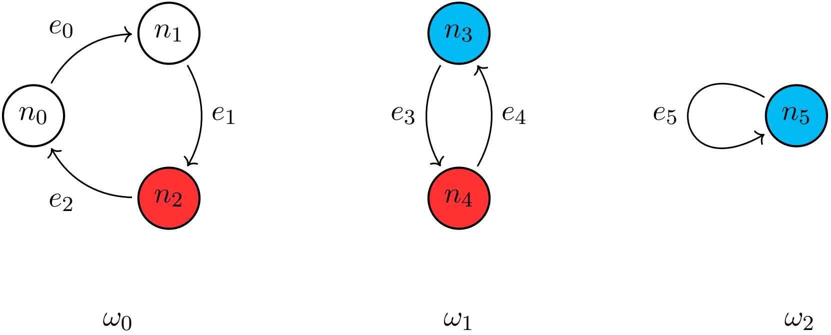
Example – QLTL



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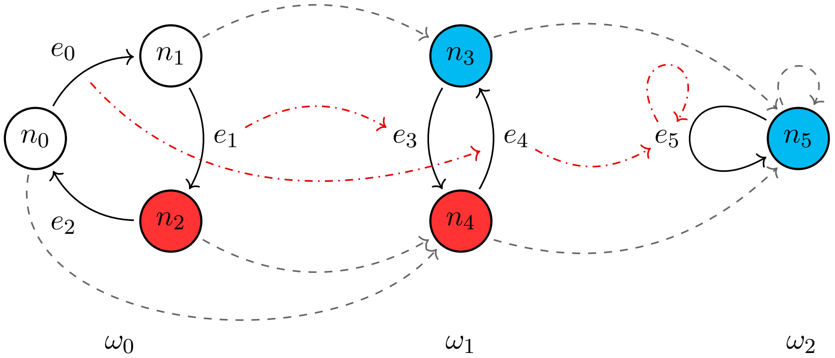

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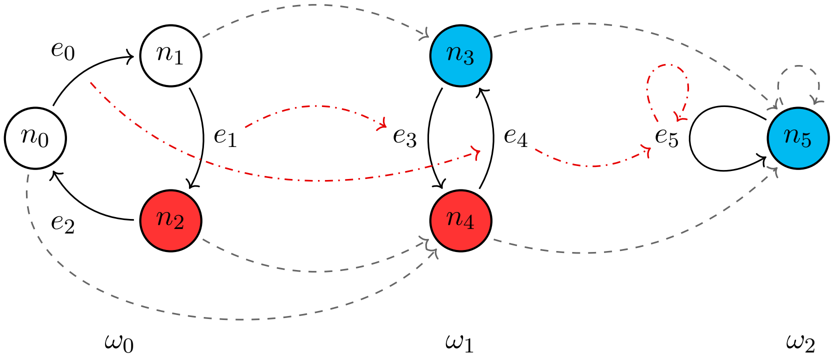

- $n_1 \vDash_{\omega_0} \mathsf{Next}(\mathsf{Blue}(x))$

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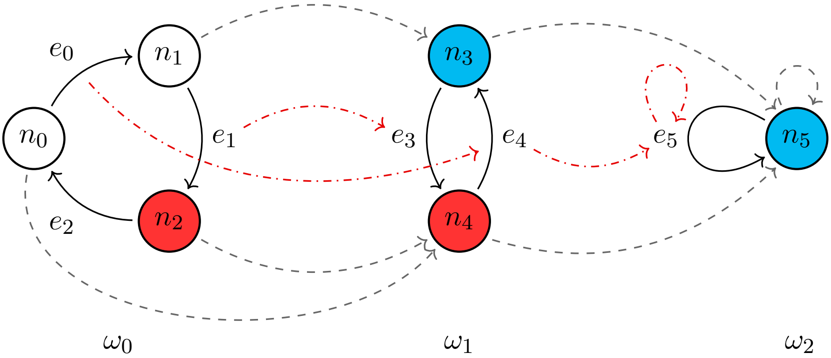

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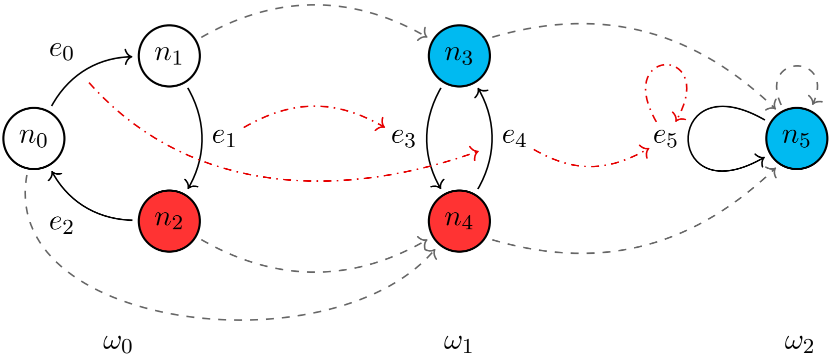

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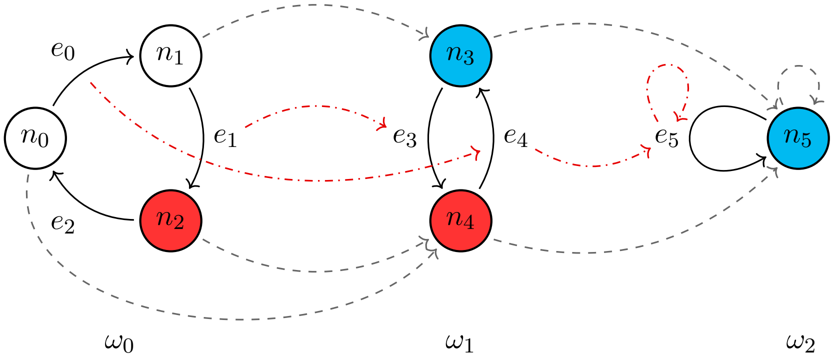

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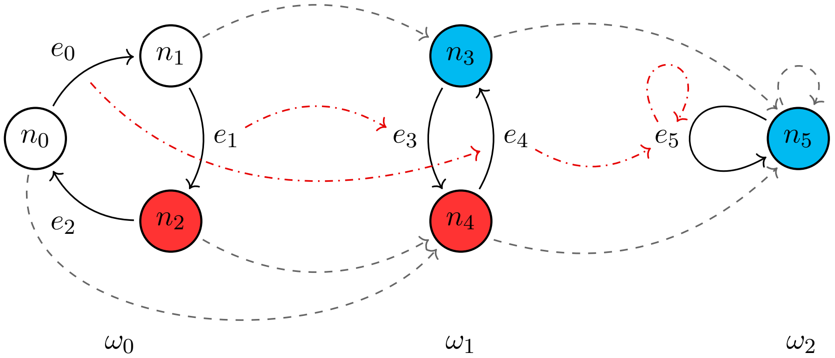

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- $() \vDash_{w_0}$ $\exists x$.Next(Blue(x))

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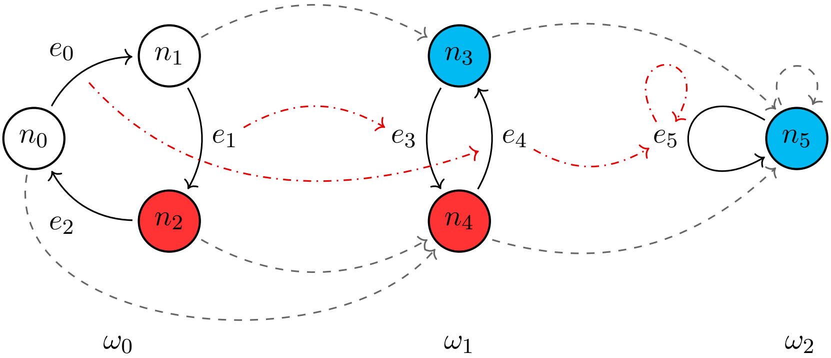

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- $(n_3, n_4) \vDash_{\omega_1} \mathsf{Next}(x = y)$
- $() \vDash_{w_0} \exists x.\mathsf{Next}(\mathsf{Blue}(x))$
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\textbf{adjacent}(x, y) &:= \exists_{\mathsf{E}} e.((s(e) =_{\mathsf{N}} x \wedge t(e) =_{\mathsf{N}} y) \\
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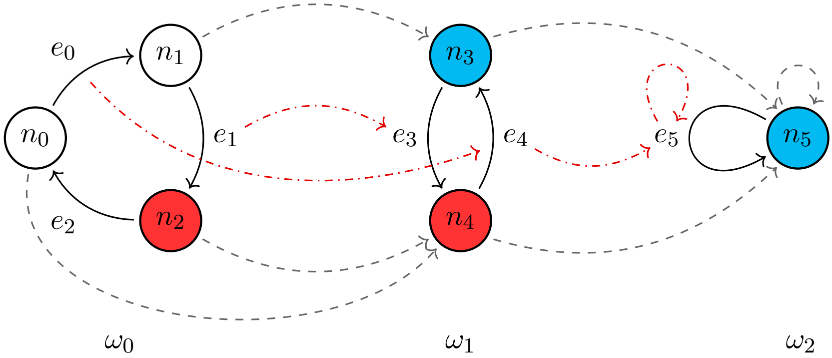
Example – QLTL on graphs



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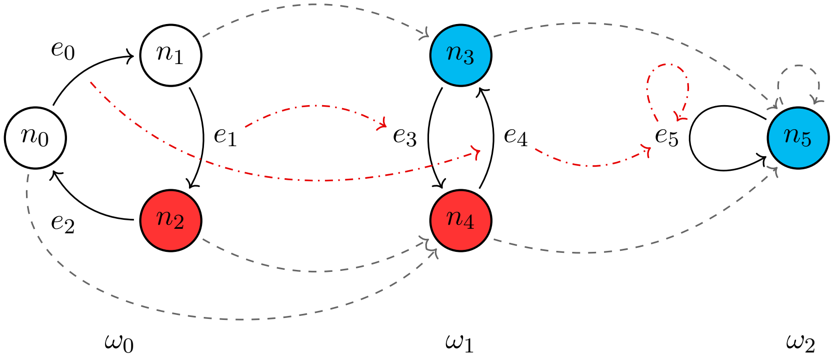

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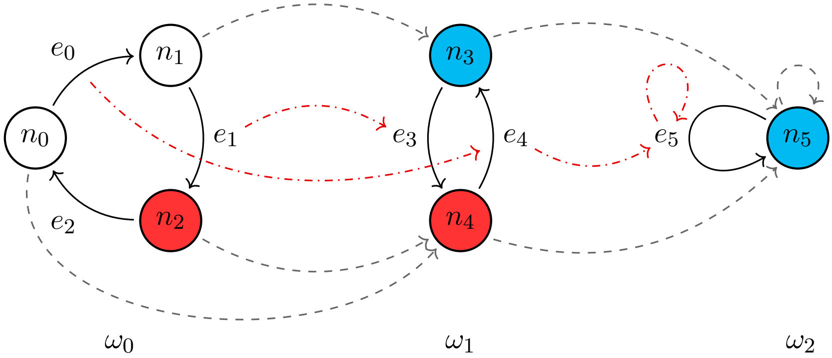

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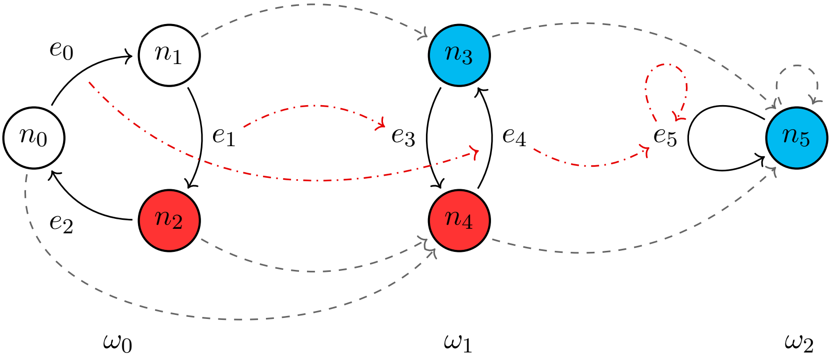

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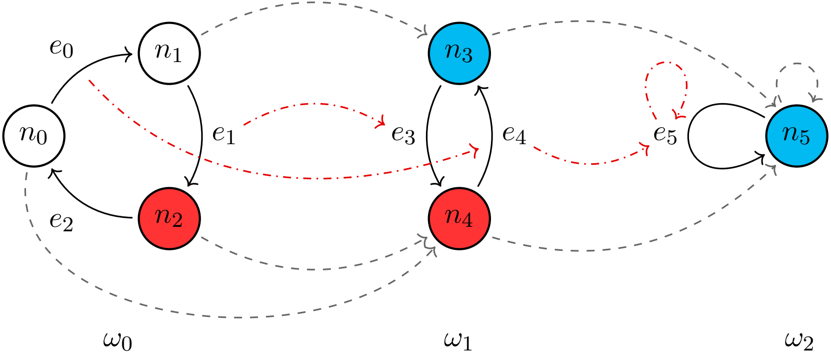

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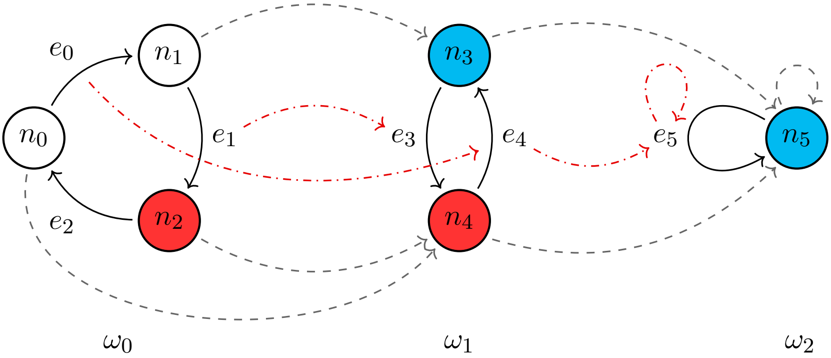

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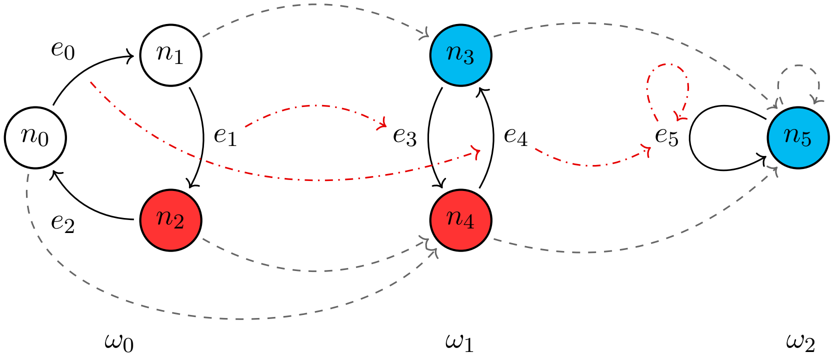

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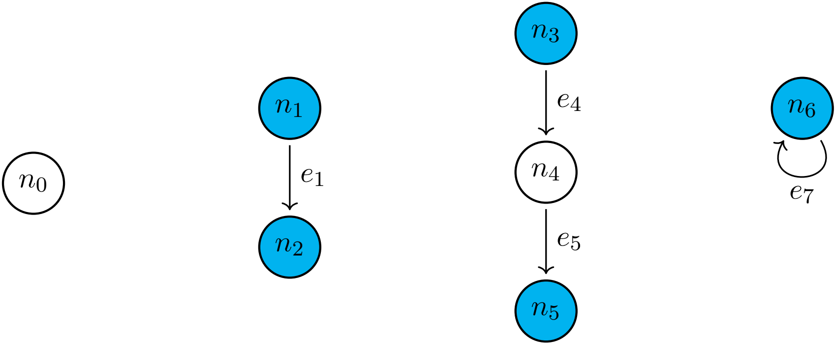
- Become particularly useful to treat duplicating relations

Example – Duplicating relations



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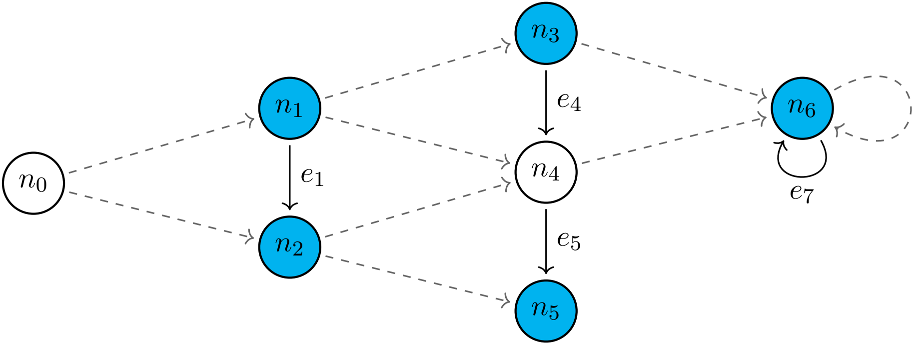

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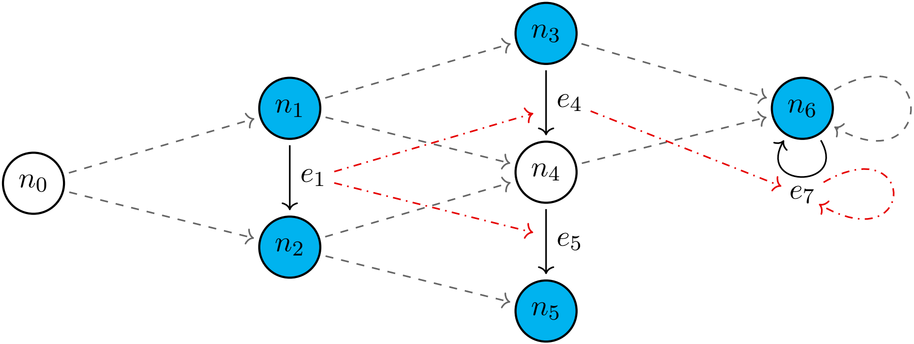

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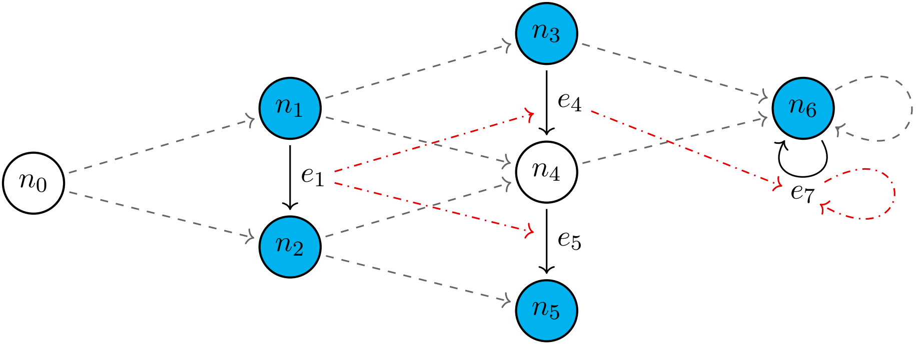

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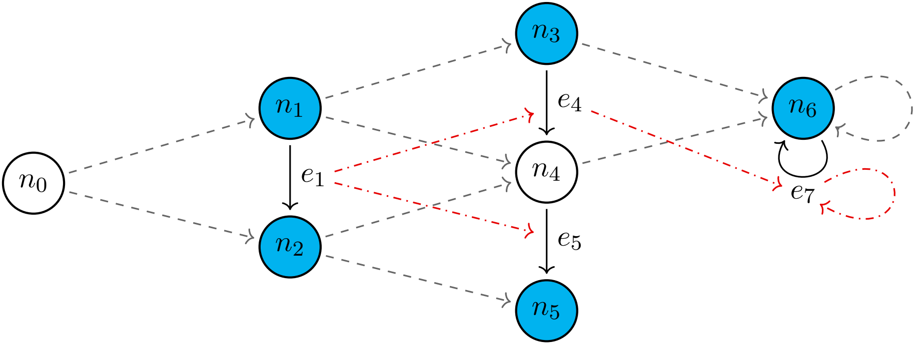

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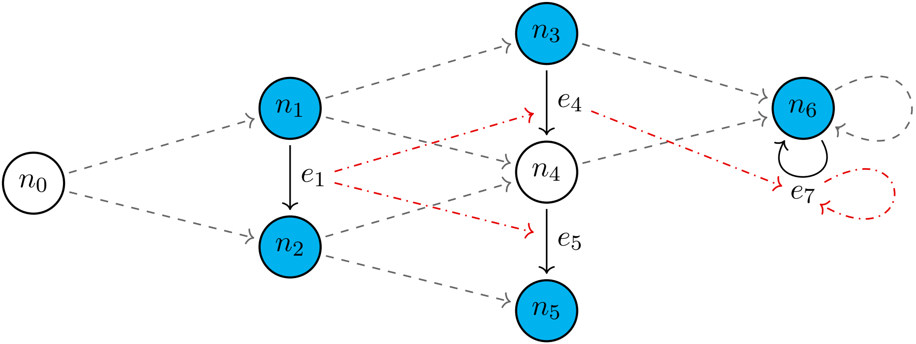
Example – Duplicating relations



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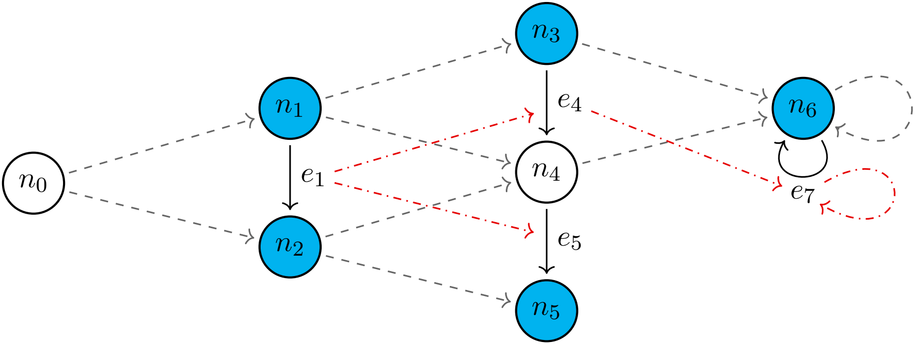

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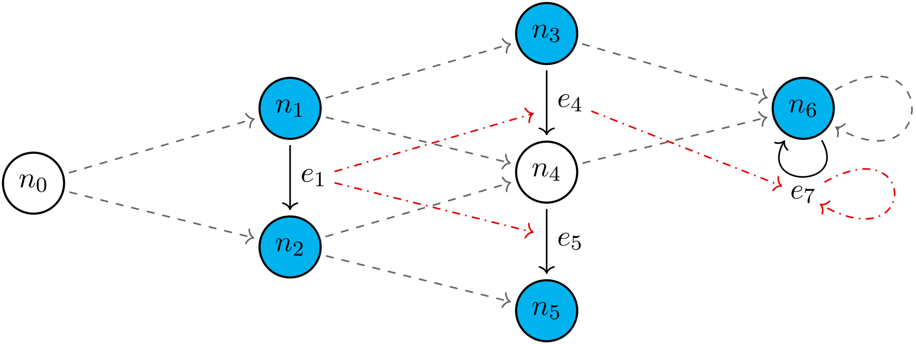

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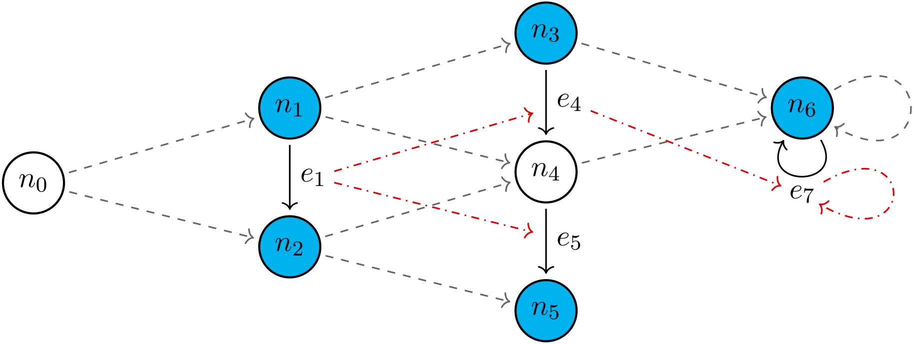

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$\Rightarrow$ (But they can be mostly recovered in the case of partial functions.)

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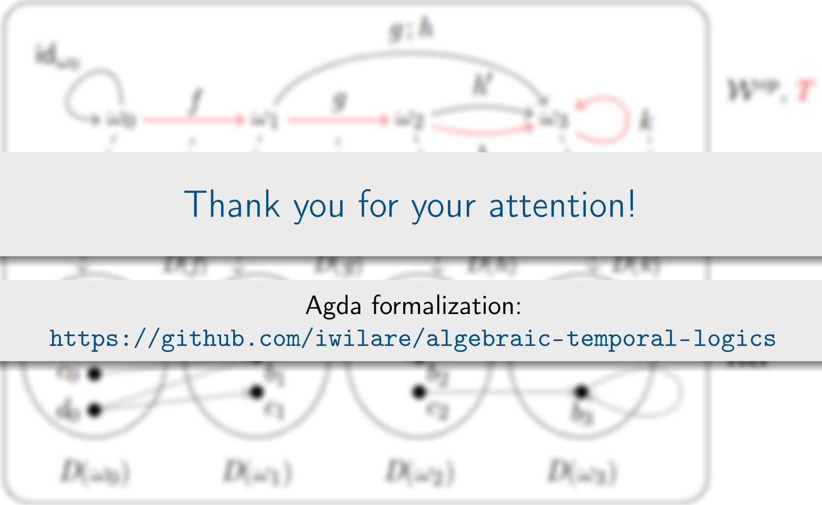

Thank you for your attention!

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